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**NATIONAL ENERGY SOFTWARE CENTER:
BENCHMARK PROBLEM BOOK**

Prepared by the
Computational Benchmark Problems Committee of the
MATHEMATICS AND COMPUTATION DIVISION
OF THE AMERICAN NUCLEAR SOCIETY

Revised June 1985

Benchmark 19
Two-dimensional PWR-Burnup Problem (with Fuel Management)

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NATIONAL ENERGY SOFTWARE CENTER:
BENCHMARK PROBLEM BOOK

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This publication is the third supplement to, and revision of, ANL-7416, originally published in February 1968, with Supplement 1 issued in December 1972 and Supplement 2 issued in June 1977.

Supplemental and replacement pages contained in this publication are as follows:

<u>PAGES TO BE INSERTED</u>	<u>PAGES TO BE REMOVED</u>	<u>COMMENT</u>
Title page	Title page	Revised page
3-6	3-6	Revised pages
9-10	9-10	Revised page
73, 73.0-73.7, 74	73-74	Revised and supplemental pages containing additional Benchmark Problem Solutions for Source Situation 4
269, 269.0-269.49, 270	269-270	Revised and supplemental pages containing additional Benchmark Problems and Benchmark Problem Solutions for Source Situation 9
589, 589.0-589.65, 590	589-590	Revised and supplemental pages containing additional Benchmark Problems and Benchmark Problem Solutions for Source Situation 14
619-924	619	Supplemental pages containing Source Situations 16 through 19 with accompanying Benchmark Problems and Benchmark Problem Solutions

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IV. BENCHMARK PROBLEMS

Source Situations

1. Small Spherical Critical Experiment
2. A High-temperature Gas-cooled Reactor Configuration
3. An Analytical Two-dimensional Multigroup Diffusion Problem
4. A Simple Highly Nonseparable Reactor
5. Two-dimensional Isolated Source in an Absorbing Medium
6. Infinite Slab Reactor Model
7. Monoenergetic Point Reactor Model
8. Two-dimensional (R-z) Reactor Model
9. Multi-dimensional (Hex-z) HTGR Model
10. PWR Thermal Hydraulics--Flow Between Two Channels With Different Heat Fluxes
11. Multi-dimensional (x-y-z) LWR Model
12. Neutron Transport in a Cylindrical 'Black' Rod
13. Neutron Transport in a BWR Rod Bundle
14. Multi-dimensional (x-y-x) BWR Model
15. Neutronic Depletion Benchmark Problems
16. Fast Reactor Kinetics
17. CANDU Kinetics
18. Fast Reactor Statics (SNR-300)
- ✓ 19. Two-dimensional PWR-Burnup Problem (with Fuel Management)

BENCHMARK SOURCE SITUATION

Identification: 19

Date Submitted:

January 1984

By: K. Koebke (KWU)

M. R. Wagner (KWU)

H.-J. Winter (KWU)

A. Wörner (IKE, Univ.
Stuttgart)

Date Accepted: October 1985

By: M. V. Gregory (SRL)

Descriptive Title:

Two-Dimensional PWR Model, Two Load Cycles

Suggested Function:

Designed to provide a test for the capabilities of
coarse mesh methods for PWR fuel management calculations

Configuration:

Two-dimensional model of a large PWR

Fig. 1 Loading Pattern of First Cycle

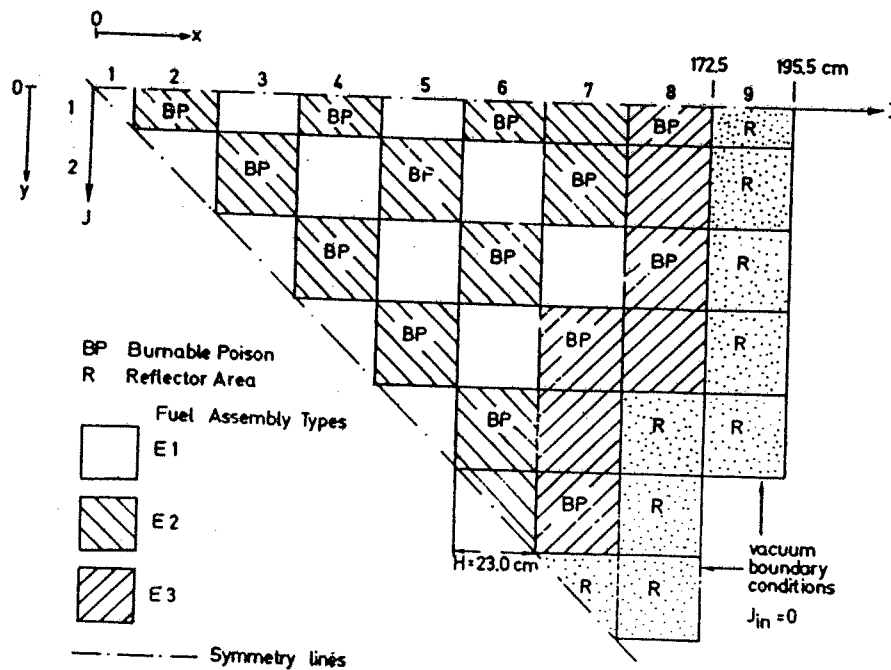
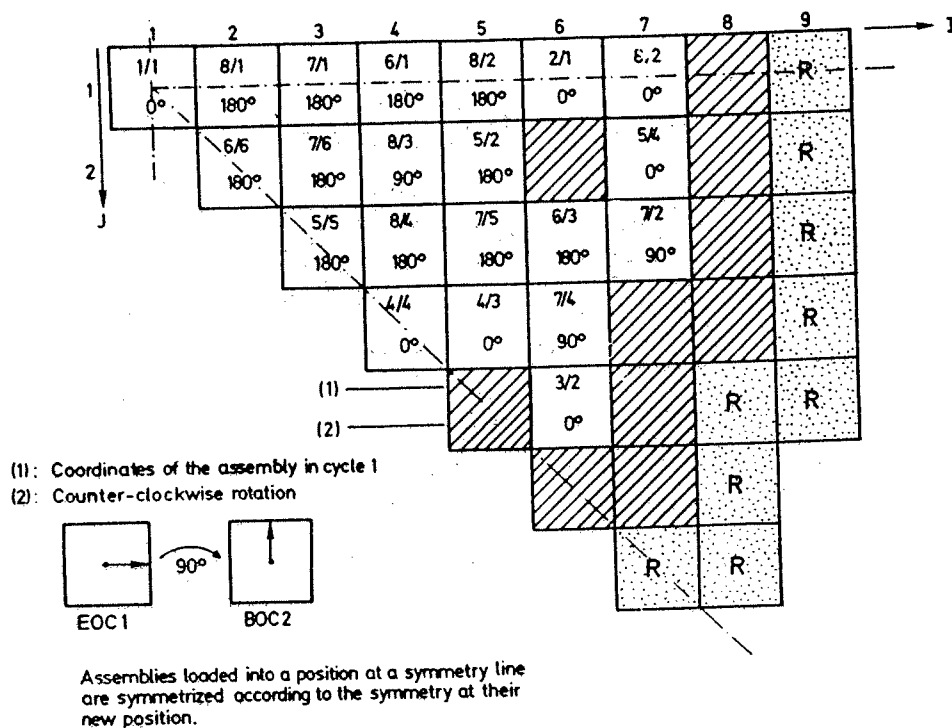


Fig. 2 Refueling Pattern of Second Cycle:



Note: The assembly which is moved from its cycle 1 position $(I, J) = (8, 2)$ to positions $(5, 1)$ and $(7, 1)$ at the main axis is to be symmetrized, at reload time and after rotating it as shown in Fig. 2, according to the following rule

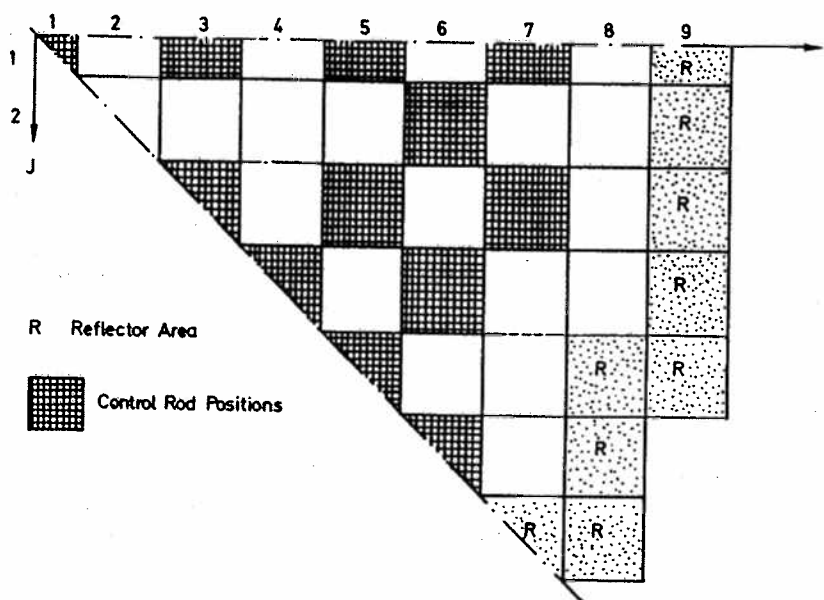
$$P_{\text{sym}}(x, y) = \frac{1}{2}(P(x, y) + P(x, -y)) ,$$

with

$$0 \leq y \leq \frac{H}{2} ,$$

where $P(x, y)$ denotes the local burnup and the local nuclide number densities, respectively.

Fig. 3 Control Rod Configuration



Details:

The problem is set up to allow the use of computer codes which model either a quarter core or a reactor octant.

Vacuum boundary conditions at external boundaries,
Reflective boundary conditions at symmetry lines.

Homogeneous reflector

Specified nuclide depletion chains
initial nuclide densities,
microscopic cross sections of burnable nuclides,
macroscopic cross sections of structural materials.

Reactor is maintained critical throughout life by adjusting the amount of soluble boron (measured in ppm).

The cycle length is defined by requiring the critical boron concentration to become equal to zero.

The selection of time steps is not specified. The benchmark problem solution should approximate as closely as possible the case of infinitely small time steps.

The isotopic depletion equations for nuclides 1 to 8 are

$$\frac{dN_j}{dt} + \left\{ \sum_g \sigma_{aj}^g \phi_g + \lambda_j \right\} N_j = N_{j-1} \sum_g \sigma_{cj-1}^g \phi_g.$$

Note: For nuclides $j=3$ (U238) and $j=8$ (burnable poison BP) the source term is zero.

For nuclide $j=9$ (Xe135) the equilibrium concentration is to be determined by

$$N_{Xe}^{\infty} = \frac{\sum_j \gamma_{Xe}^j N_j \sum_g \sigma_{fj}^g \phi_g}{\sum_g \sigma_{cXe}^g \phi_g + \lambda_{Xe}}$$

The depletion equation of the fission product (FP) $j=10$ has no loss term:

$$\frac{dN_{FP}}{dt} = \sum_{j=1}^7 \gamma_{FP}^j N_j \sum_g \sigma_{fj}^g \phi_g$$

Table 1. Initial Nuclide Densities (Units of 10^{24} atoms/cm³):

Nuclide	Fuel Assembly Type		
	E1	E2	E3
U235	1.40 E-4	1.60 E-4	2.20 E-4
U238	6.31 E-3	6.29 E-3	6.23 E-3
BP	0.	8.00 E-6 a)	8.00 E-6 a)

All other depletable nuclides have zero initial number densities

a) where present according to the load pattern of cycle 1,
Fig. 1

BENCHMARK PROBLEM

Identification: 19-A1

Source Situation: ID.19

Date Submitted:
January 1984By: K. Koebke (KWU)
M. R. Wagner (KWU)
H.-J. Winter (KWU)
A. Wörner (IKE, Univ. Stuttgart)

Revised June 1984

Date Accepted: October 1985 By: M. V. Gregory (SRL)

Descriptive Title:

Two-Dimensional PWR Burnup Problem

Reduction of Source Situation:

1. Diffusion theory
2. Two energy groups
3. Zero axial leakage
4. Reactor is maintained critical at all times by adjusting the amount (ppm) of soluble boron (except for the rodged reactor at EOC1, see point 7 below)
5. The reactor is at all times in its (instantaneous) xenon equilibrium state
6. In addition, the critical state of the xenon-free reactor is to be determined at BOC1, EOC1, and BOC2 (see below)
7. The control rods are withdrawn during depletion. At the beginning of cycles 1 and 2 the following procedure should be followed for determining the rod worth and the power distribution for the rodged reactor:
 - a. Compute the critical boron concentration c_{BOC}^0 (un-rodged) for the unrodged, xenon-free reactor
 - b. Insert the control rods and compute the critical boron concentration c_{BOC}^0 (rodged) and the normalized power distribution for the rodged, xenon-free core

At end of cycle 1 the procedure is

- a. Compute c_{EOC}^0 (unrodded) for the xenon-free, unrodded reactor.
- b. Keep the soluble boron fixed at c_{EOC}^0 (unrodded) and compute the reactivity of the rodded, xenon-free reactor.

The non-linear two-group diffusion equations to be solved are

$$\begin{aligned}
 -\nabla D^1 \nabla \phi_1 + \Sigma_r^1 \phi_1 &= \sum_j N_j \sum_g v_j^g \sigma_{fj}^g \phi_g, \\
 -\nabla D^2 \nabla \phi_2 + \Sigma_r^2 \phi_2 &= \Sigma^{12} \phi_1.
 \end{aligned}$$

where ppm is the eigenvalue ^{*)} and the following cross section definitions apply

$$\Sigma_r^1 = \Sigma^{12} + \Sigma_c^1 + \Sigma_f^1 + \text{ppm} \cdot \sigma_{c,SB}^1$$

$$\Sigma_r^2 = \Sigma_c^2 + \Sigma_f^2 + \text{ppm} \cdot \sigma_{c,SB}^2 + \sigma_{c,Xe}^2 N_{Xe}^\infty$$

$$D^g = 1/(3\Sigma_{tr}^g),$$

Note: The macroscopic cross sections Σ^g include contributions from the structural material Σ_{STRM} . For the rodded reactor there are additional contributions $\Delta\Sigma_R$ from the control rods for all rodded assemblies of Fig. 3. These contributions are indicated by brackets.

^{*)} with the exception of the rodded reactor at EOC1 where k_{eff} is the eigenvalue

Transport cross sections

$$\Sigma_{tr}^g = \Sigma_{tr,STRM}^g$$

Fission

$$\Sigma_f^g = \sum_{j=1}^7 \sigma_{fj}^g N_j$$

Capture (not including soluble boron and xenon)

$$\Sigma_c^g = \sum_{j \neq 9, 11} \sigma_{cj}^g N_j + \Sigma_{c,STRM}^g + (\Delta \Sigma_{c,R}^g)$$

Slowing down

$$\Sigma^{12} = \Sigma_{STRM}^{12} + (\Delta \Sigma_R^{12})$$

The two-group cross sections and other data are listed in the following tables:

Table 1: Microscopic Cross Sections of Fissionable Nuclides

Table 2: Microscopic Cross Sections of Nuclides 8 to 11

Table 3: Fission Product Yields

Table 4: Macroscopic Cross Sections of the Structural Material

Table 5: Macroscopic Cross Sections of the Control Rods

Table 1. Microscopic Cross Sections of Fissionable Nuclides (unit 10^{-24} cm^2)

Nuclide Group		Assembly Type						Neutrons per Fission ν_j^g
j	g	E1		E2		E3		
		σ_f^g	σ_c^g	σ_f^g	σ_c^g	σ_f^g	σ_c^g	
1 U235	1	7.60	4.20	7.50	4.10	7.40	4.00	2.44
	2	238.60	42.90	231.90	41.80	220.60	39.80	2.44
2 U236	1	0.20	7.90	0.20	7.80	0.20	7.70	2.79
	2	0.00	2.70	0.00	2.70	0.00	2.60	0.00
3 U238	1	0.12	0.85	0.12	0.84	0.12	0.84	2.85
	2	0.00	1.23	0.00	1.20	0.00	1.15	0.00
4 Pu239	1	9.80	6.40	9.70	6.40	9.70	6.40	2.90
	2	617.00	348.10	611.60	347.00	601.50	344.50	2.88
5 Pu240	1	0.60	10.00	0.60	10.00	0.60	10.00	3.10
	2	0.10	915.90	0.10	977.50	0.10	1081.20	2.89
6 Pu241	1	18.50	4.70	18.40	4.70	18.20	4.70	3.02
	2	635.20	222.80	622.30	218.90	599.90	212.00	2.94
7 Pu242	1	0.50	36.50	0.50	36.20	0.50	35.70	3.11
	2	0.00	9.90	0.00	9.80	0.00	9.50	0.00

κ_j^g , the energy released per fission event is $\kappa_j^g = 0.32 \text{ E-10 Ws}$
for all fissionable nuclides j, groups g and assembly types.

Table 2.

Microscopic Capture Cross Sections σ_c of Nuclides 8 to 11,
units 10^{-24} cm^2 for nuclides 8-10,
unit $\text{cm}^{-1} \text{ ppm}^{-1}$ for nuclide 11.

Nuclide j		Group g	σ_c^g (All Assembly Types)
8	BP	1	28.00
		2	800.0
9	Xe135	1	0.0
		2	1.20E+6
10	FP	1	5.3
		2	36.5
11	SB *)	1	1.40E-7
		2	9.38E-6

*) note: units for nuclide 11 are $\text{cm}^{-1} \text{ ppm}^{-1}$

Table 3 Fission Product Yields γ_k^j (All Assembly Types)

Nuclide k \ j	U235 1	U236 2	U238 3	Pu239 4	Pu240 5	Pu241 6	Pu242 7
	1	2	3	4	5	6	7
9 Xe135	.0663	.0665	.0665	.0747	.0665	.0708	.0665
10 FP	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 4. Macroscopic Cross Sections (cm^{-1}) of the Structural Material STRM

Note: $D_g = 1.0 / (3\Sigma_{tr}^g)$

Assembly Type	Group g	Σ_c^g, STRM	$\Sigma_{tr}^g, \text{STRM}$	$\Sigma_{\text{STRM}}^{12}$
E1	1	6.51E-4	0.222	1.89E-2
	2	7.02E-3	0.835	
E2	1	6.50E-4	0.222	1.88E-2
	2	6.93E-3	0.829	
E3	1	6.50E-4	0.222	1.85E-2
	2	6.78E-3	0.820	
Reflector	1	9.80E-4	0.257	2.37E-2
	2	0.138	1.31	

Table 5. Macroscopic Cross Sections of the Control Rods, all assembly types

Group g	$\Delta\Sigma_c^g, R$	$\Delta\Sigma_R^{12}$
1	0.002	-0.003
2	0.020	-

Additional Data:

Average power density is 93 Watt/cm³ at all times (averaged over entire octant core) resp. 1.186877625E+6 W/cm for the reactor octant.

The burnup (exposure) is expressed in units of MWD/kgU.

The uranium mass density in all assembly types is 2.54E-3 kg/cm³.

Expected Primary Results

For reactor cycles 1 and 2:

1. Critical boron concentration (ppm) at discrete time points
2. Average normalized assembly powers at BOC and EOC
3. Average assembly burnups at EOC
4. Average nuclide densities for each fuel batch at EOC
5. Cycle length in units of full power days
6. Critical boron concentration and normalized power distribution for the rodded, xenon-free reactor (BOC1 and BOC2)
7. Reactivity of the rodded, xenon-free reactor (EOC1)
8. Total running time and computer used

Suggested Additional Results

1. Maximum power per assembly and location of maximum in local coordinates for each assembly at BOC1 and BOC2 *)
2. Maximum value of burnup and its location in local coordinates for each assembly at EOC1 and EOC2
3. Dependence of results and computing time on
 - a. spatial mesh for flux calculation
 - b. spatial discretization of burnup calculation
 - c. time step lengths.

*) It is recommended to compute the maximum values for a local grid of 16x16 (homogenized) "pins" for each assembly. The maximum value of the average "pin powers" per assembly and its location in terms of local pin coordinates is the suggested result.

BENCHMARK PROBLEM SOLUTION

Identification: 19-A1-1

Benchmark Problem: ID.19-A1

Date Submitted: August 1984

By: H.-J. Winter (KWU)
K. Koebke (KWU)
M. R. Wagner (KWU)

Date Accepted: October 1985

By: M. V. Gregory (SRL)

Problem:

Two-dimensional PWR Burnup problem⁴

Mathematical Models:

1. Nodal diffusion calculation

The nodal expansion method (NEM)¹ with polynomials of 4. order, quadratic transverse leakage approximation¹ and burnup-correction⁵ was used with a square mesh of $h = 4.6$ cm, which corresponds to 5x5 nodes per assembly. Explicit use was made of the octant symmetry so that the total number of nodes is equal to 841.

2. Burnup and control rod calculation

The depletion calculation was done for Xe-equilibrium states and for the following time steps (in full power days) between diffusion calculations

Cycle 1: 1.5(4), 12(2), 15(23), 6.49, 10^{-6} daysCycle 2: 1.5(4), 12(2), 15(15), 7.04, 10^{-6} days

total number of time steps 54

total number of diffusion calculations 62

The control rods are withdrawn during depletion. Control rod worths are determined for the xenon-free reactor at BOC 1 and BOC 2 in ppm, at EOC 1 in % reactivity

Computing time: 1650 CP-sec (total)
Computer: CYBER 176
Code: MEDIUM-2^{2,5}

3. Determination of maximum powers and burnups

At BOC and EOC data files were saved which contained the converged nodal variables (node averaged fluxes and face averaged partial currents) and the average burnups and the macroscopic cross sections per node. The editing code PINPOW was then used to reconstruct the two-dimensional flux and power distributions within each fuel assembly. For each node a high order local interpolation method³ was applied to determine the peak powers. The location of the maximum values of "pin" powers and burnups per assembly is given in terms of local pin coordinates for a 16x16 array of square "pins" for each assembly, see tables 3a, 6a, 8a, 9a.

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ID.19-A1-1

Results

Tab. 1:	Critical boron concentrations vs. core averaged burnup	E
Tab. 2:	Average normalized assembly powers at BOC 1 (without Xe)	E
Tab. 3:	Average normalized assembly powers at BOC 1 (Xe-equilibrium)	E
Tab. 3a:	Maximum powers per assembly at BOC 1 (Xe-equilibrium)	A
Tab. 4:	Average normalized assembly powers at EOC 1 (Xe-equilibrium)	E
Tab. 5:	Average normalized assembly powers at BOC 2 (without Xe)	E
Tab. 6:	Average normalized assembly powers at BOC 2 (Xe-equilibrium)	E
Tab. 6a:	Maximum powers per assembly at BOC 2 (Xe-equilibrium)	A
Tab. 7:	Average normalized assembly powers at EOC 2 (Xe-equilibrium)	E
Tab. 8:	Average assembly burnups at EOC 1	E
Tab. 8a:	Maximum values of burnups at EOC 1	A

Tab. 9:	Average assembly burnups at EOC 2	E
Tab. 9a:	Maximum values of burnups at EOC 2	A
Tab. 10:	Average nuclide densities for each fuel batch at EOC 1	E
Tab. 11:	Average nuclide densities for each fuel batch at EOC 2	E
Tab. 12:	Control rod worths at BOC 1, EOC 1, BOC 2	E
Tab. 13:	Average normalized assembly powers at BOC 1, A all control rods inserted (no Xe)	A
Tab. 14:	Average normalized assembly powers at BOC 2, A all control rods inserted (no Xe)	A

Note: E means expected primary results; A means suggested additional results; BOC beginning of cycle; EOC end of cycle

CYCLE 1

Time Point t (efpd)	Core Average Burnup B (MWd/kgU)	Critical Boron Concentration C (ppm)
0.	0.	1277.8 (without Xe)
0.	0.	1038.6 (Xe equilibrium)
30.	1.098	1081.7
60.	2.197	1069.2
90.	3.295	1019.0
120.	4.394	944.1
150.	5.492	853.1
180.	6.590	751.5
210.	7.689	643.5
240.	8.787	531.7
270.	9.886	418.1
300.	10.984	304.3
330.	12.082	191.0
360.	13.181	79.1
381.49	13.968	0.

CYCLE 2

0.	8.754	1260.6 (without Xe)
0.	8.754	979.5 (Xe equilibrium)
30.	9.853	891.5
60.	10.951	789.0
90.	12.050	677.5
120.	13.148	560.9
150.	14.246	442.0
180.	15.345	322.5
210.	16.443	203.4
240.	17.542	85.5
262.04	18.349	0.

Tab.1

Boron concentrations vs. core average burnup,
efpd = equivalent full power days per cycle

	1	2	3	4	5	6	7	8
1	1.114	1.128	1.107	1.116	1.092	1.106	1.137	.863
2		1.110	1.121	1.096	1.103	1.074	1.036	.896
3			1.097	1.103	1.075	1.080	.996	.782
4				1.070	1.068	1.050	1.158	.639
5					1.009	.964	.916	
6						.816	.503	

Tab. 2

Average normalized assembly powers at BOC 1, no Xe

	1	2	3	4	5	6	7	8
1	1.164	1.175	1.151	1.150	1.115	1.107	1.113	.827
2		1.157	1.161	1.130	1.122	1.078	1.016	.859
3			1.135	1.131	1.093	1.077	.975	.751
4				1.093	1.077	1.041	1.114	.612
5					1.011	.950	.882	
6						.802	.490	

Tab. 3Average normalized assembly powers at BOC 1,
Xe-equilibrium

	1	2	3	4	5	6	7	8
1	1.194 8 8	1.222 1 16	1.181 7 8	1.201 1 16	1.145 7 8	1.158 1 16	1.170 5 8	1.187 1 9
2		1.187 8 8	1.215 1 1	1.161 7 8	1.181 1 1	1.111 6 7	1.121 1 1	1.186 1 2
3			1.166 7 7	1.191 1 1	1.124 6 7	1.142 1 1	1.040 4 11	1.127 1 1
4				1.125 7 6	1.148 1 1	1.079 9 6	1.364 1 1	1.043 1 1
5					1.053 5 5	1.071 1 1	1.239 1 1	
6						.968 1 1	.954 1 1	

Tab. 3 a

BOC 1, Xe-equilibrium

Maximum powers per assembly (pin averages) and location of maximum in local pin-coordinates

	1	2	3	4	5	6	7	8
1	1.036	1.083	1.038	1.089	1.046	1.095	1.057	.868
2		1.037	1.086	1.042	1.094	1.045	1.052	.872
3			1.041	1.093	1.050	1.095	.996	.833
4				1.050	1.101	1.044	1.117	.673
5					1.047	1.055	.923	
6						.927	.634	

Tab. 4

Average normalized assembly powers at EOC 1,
Xe-equilibrium

	1	2	3	4	5	6	7	8
1	.699	.947	.786	.800	1.059	.885	1.082	.971
2		.831	1.082	1.076	.862	1.345	.827	.921
3			.814	1.149	1.047	.835	.839	.907
4				.753	.826	1.065	1.402	.786
5					1.458	1.014	1.225	
6						1.500	.888	

Tab. 5

Average normalized assembly powers at BOC 2, no Xe

	1	2	3	4	5	6	7	8
1	.735	.985	.816	.824	1.077	.898	1.085	.969
2		.865	1.111	1.096	.878	1.353	.835	.920
3			.838	1.165	1.056	.842	.839	.899
4				.768	.830	1.050	1.365	.771
5					1.431	.990	1.181	
6						1.440	.853	

Tab. 6Average normalized assembly powers at BOC 2,
Xe-equilibrium

	1	2	3	4	5	6	7	8
1	.748 15 15	1.061 1 16	.841 3 16	.846 15 13	1.135 1 16	.957 8 16	1.198 16 8	1.232 2 8
2		.881 3 3	1.146 1 16	1.135 16 1	.953 16 8	1.440 8 8	.929 1 6	1.187 2 1
3			.871 16 3	1.254 3 3	1.198 1 3	.931 6 1	.937 7 16	1.200 1 16
4				.828 1 1	.952 12 16	1.214 16 10	1.484 5 11	1.236 1 4
5					1.541 11 11	1.085 1 16	1.454 4 1	
6						1.561 6 6	1.394 1 2	

Tab. 6 a

BOC 2, Xe-equilibrium

Maximum powers per assembly (pin averages) and location of maximum in local pin-coordinates

	1	2	3	4	5	6	7	8
1	.935	1.168	.972	.946	1.114	.937	1.071	.969
2		1.030	1.234	1.165	.941	1.302	.881	.944
3			.941	1.172	1.071	.885	.869	.912
4				.842	.866	1.001	1.187	.748
5					1.259	.893	.998	
6						1.152	.737	

Tab. 7

Average normalized assembly powers at EOC 2,
Xe-equilibrium

	1	2	3	4	5	6	7	8
1	15.181	15.757	15.189	15.753	15.125	15.531	15.011	11.641
2		15.186	15.762	15.175	15.674	14.869	14.483	11.838
3			15.184	15.725	15.058	15.354	13.704	10.866
4				15.081	15.472	14.616	15.503	8.764
5					14.506	14.133	12.438	
6						11.984	7.665	

Tab. 8

Average assembly burnups (MWd/kgU) at EOC 1

	1	2	3	4	5	6	7	8
1	15.296 9 9	16.079 16 16	15.305 9 9	16.080 1 16	15.258 6 9	15.942 1 16	15.620 3 9	15.997 1 16
2		15.303 9 9	16.082 1 16	15.297 6 8	16.051 1 1	15.090 3 6	15.530 1 1	15.971 1 3
3			15.302 8 8	16.075 1 1	15.209 6 6	15.873 1 1	14.479 2 14	15.584 1 1
4				15.232 6 6	15.967 1 1	14.919 5 3	18.380 1 1	14.405 1 1
5					14.868 4 3	15.406 1 1	16.903 1 1	
6						14.236 1 1	14.038 1 1	

Tab. 8 a

EOC 1

Maximum values of burnups per assembly (pin averages) and location of maximum in local pin-coordinates.

	1	2	3	4	5	6	7	8
1	22.990	21.741	23.384	23.827	22.191	24.554	22.278	9.574
2		20.863	18.677	21.470	24.271	12.840	23.792	9.212
3			22.811	19.686	22.456	23.611	22.769	8.950
4				22.625	23.757	25.331	12.488	7.490
5					12.948	24.837	10.660	
6						12.621	7.780	

Tab. 9

Average assembly burnups (MWd/kgU) at EOC 2

	1	2	3	4	5	6	7	8
1	23.155 14 14	26.030 16 16	23.896 15 14	24.240 16 16	26.179 16 16	25.141 8 16	26.116 1 16	11.916 2 8
2		22.947 15 15	24.622 16 16	25.985 1 16	25.050 16 8	13.441 8 8	24.805 1 4	11.654 1 1
3			23.168 14 14	24.105 16 16	26.132 16 16	24.494 8 1	24.256 5 16	11.507 1 16
4				23.155 3 3	24.730 10 16	28.324 1 16	13.323 6 10	11.656 1 4
5					13.629 11 10	25.744 1 16	12.993 4 1	
6						13.515 6 6	12.380 1 1	

Tab. 9 a

EOC 2

Maximum values of burnups per assembly (pin averages) and location of maximum in local pin-coordinates

Nuclide Densities ($10^{24}/\text{cm}^3$) per Fuel Batch

Nuclide	E1	E2	E2+BP	E3	E3+BP	Core Average
U235	.62936E-04	.83668E-04	.76318E-04	.14813E-03	.14662E-03	.91245E-04
U236	.13320E-04	.13383E-04	.14627E-04	.13042E-04	.13335E-04	.13751E-04
U238	.62322E-02	.62242E-02	.62145E-02	.61846E-02	.61824E-02	.62125E-02
PU239	.25942E-04	.24890E-04	.26137E-04	.22059E-04	.22445E-04	.24977E-04
PU240	.76418E-05	.63720E-05	.72203E-05	.40813E-05	.42756E-05	.65095E-05
PU241	.46266E-05	.36803E-05	.45508E-05	.19388E-05	.21425E-05	.38661E-05
PU242	.87955E-06	.57834E-06	.80167E-06	.20114E-06	.24241E-06	.66283E-06
BP	.0	.0	.97646E-06	.0	.25601E-05	.71545E-06
XE135	.19091E-08	.20080E-08	.20379E-08	.22478E-08	.22714E-08	.20533E-08
FP	.10167E-03	.92564E-04	.10499E-03	.75527E-04	.78092E-04	.95790E-04

Tab. 10

Average nuclide densities for each fuel batch at EOC 1

Nuclide Densities ($10^{24}/\text{cm}^3$) per Fuel Batch

Nuclide	E1	E2	E2+BP	E3	E3+BP	E3 ^{*)}	Core Average
U235	.39932E-04	.53514E-04	.48659E-04	.97886E-04	.97036E-04	.15217E-03	.98062E-04
U236	.16758E-04	.18114E-04	.18836E-04	.21539E-04	.21705E-04	.12313E-04	.17164E-04
U238	.61861E-02	.61774E-02	.61672E-02	.61373E-02	.61348E-02	.61883E-02	.61675E-02
PU239	.28511E-04	.28530E-04	.28878E-04	.29153E-04	.29417E-04	.21144E-04	.26234E-04
PU240	.10037E-04	.92252E-05	.96091E-05	.79444E-05	.80612E-05	.37135E-05	.71044E-05
PU241	.78787E-05	.74089E-05	.80704E-05	.61410E-05	.63093E-05	.16405E-05	.52732E-05
PU242	.25953E-05	.21214E-05	.25429E-05	.12624E-05	.13447E-05	.15097E-06	.13521E-05
BP	.0	.0	.27214E-06	.0	.75097E-06	.0	.18791E-06
XE135	.18035E-08	.19451E-08	.18835E-08	.23455E-08	.23630E-08	.23789E-08	.21839E-08
FP	.15601E-03	.15172E-03	.16383E-03	.14717E-03	.14960E-03	.70187E-04	.12583E-03

Tab. 11

Average nuclide densities for each fuel batch at EOC 2

*) Inserted at BOC 2.

Control Rod Worths

Time Point	C _O (ppm) unrodded	C _R (ppm) rodded	control rod worths in Δc (ppm) resp. $\Delta \rho_{eff}$ (%)
BOC 1	1277.8	530.1	747.7 ppm
EOC 1	248.2	248.2	-8.078 %
BOC 2	1260.	526.4	734.2 ppm

Tab. 12

Control rod worths at BOC 1, EOC 1 and BOC 2 for Xenon-free reactor state; $\rho_{eff} = (k_{eff} - 1)/k_{eff}$, k_{eff} eigenvalue

	1	2	3	4	5	6	7	8
1	2.270	3.251	1.852	2.114	.906	.720	.515	.692
2		3.366	2.620	2.138	1.233	.508	.659	.770
3			1.400	1.427	.651	.611	.446	.633
4				.637	.615	.422	.805	.571
5					.372	.557	.732	
6						.347	.358	

Tab. 13

BOC 1

Average normalized assembly powers, all control rods inserted,
no Xe

	1	2	3	4	5	6	7	8
1	.696	1.263	.667	.772	.545	.540	.799	1.478
2		1.144	1.233	1.071	.594	.634	.858	1.528
3			.573	.904	.514	.607	.757	1.583
4				.361	.505	.744	2.064	1.501
5					.724	.988	1.856	
6						1.084	1.122	

Tab. 14

BOC 2

Average normalized assembly powers, all control rods inserted,
no Xe

BENCHMARK PROBLEM SOLUTION

Identification: 19-A1-2 Benchmark Problem: ID.19-A1

Date Submitted: February 1985 By: D. R. Vondy (ORNL)

Date Accepted: October 1985 By: M. V. Gregory (SRL)

Descriptive Title: Two-Dimensional PWR-Burnup Problem With Fuel Management Treating Two Cycles¹

Mathematical Model

The problem was solved with several models on a local analysis system.² The mesh-centered finite-difference spatial representation of the diffusion-theory transport approximation was applied with the VENTURE neutronics code at specific points in time. The two-dimensional quarter-core neutronics problems were solved by direct iteration toward the critical soluble boron concentration. The effect of operation at power was modeled with the BURNER code, applying a matrix exponential technique to solve the coupled chain equations with flux renormalization in substeps to maintain the desired power level of 2.3737552 ± 6 watts for a 1-cm-thick (93 W/cm^3) quarter core.

A two-cycle reactor-history calculation was done in one pass on the computer, often along with rod-insertion calculations. Lacking automated capability for establishing the end-of-cycle time and conditions, the calculations were done for specified exposure times and then redone to effect accurate results. To model the end-of-cycle state, the soluble boron concentration was set to zero and k was calculated. A useful approximation of $0.00037 \Delta k/\text{day}$ was used in predicting changes and interpreting results. Also in reporting results, the net fissile consumption was assumed to be proportional to the exposure time. A maximum relative-point flux change on outer iteration of 0.0005 was specified as criterion for terminating iteration (convergence level), typical of application to this kind of neutronics problem but somewhat relaxed from what has been used for benchmarking. The difference in the value of k from unity at the end of each cycle for the reported results was usually less than ± 0.00005 (cycle time ± 0.01 days).

Details

Each interval for exposure was divided into subintervals, and flux-level adjustments were made after each of these to effect the desired power level on the average over an exposure step. Calculations were done: (a) **marchout**, with successive neutronics and exposure calculations; or (b) **redepletion**, wherein a long-exposure period was used for an initial calculation and then the exposure was redone with the average of the start- and end-of-period flux values. Generally, equilibrium ^{135}Xe was calculated with the neutronics code but using macroscopic data and total zone-average fission rates. A macroscopic yield fraction for ^{135}Xe of 0.0690 was used excepting the values of 0.06631 for the clean core and 0.0698 at the end of each cycle before refueling. Note that the use of a high-yield value for freshly loaded fuel should cause the ^{135}Xe concentration to be overestimated suppressing the thermal flux somewhat and tending to underestimate the power density there. (A few cases were run to generate information about the ^{135}Xe feedback effects allowing the error in the reported results to be examined, but as the discrepancy was found

to be small such results are not reported.) All redepletion cases were done with a 2-day exposure period treated explicitly at the end of each cycle to effect a good representation of equilibrium ^{135}Xe .

The intent in this effort was to generate good results independently from the problem originator. Key objectives included

1. producing results for specific modelings using finite difference demonstrating their dependence (sensitivity to) to the model variables, allowing discretization error evaluation,
2. produce a relatively accurate solution; and
3. produce benchmark-quality results for the problem by reliable discretization-error removal.

There are three discretization error sources associated with the number of depletion zones, the number of meshpoints, and the discretization of time. The numbers used are shown here (those emphasized are in bold):

Number of zones (octant): 31, **102**, **397**, 1566
Number of meshpoints: 81, 289, **1156**, **4629**, 18496
Number of time steps (first cycle, second cycle):
(**4,3**), (**8,6**), (12,9) marchout;
2(1,1), 2(2,2), **2(3,3)**, 2(4,4) redeplete.

Note that with only 31 depletion zones (one per fuel element) the effects of fuel-element rotation are not modeled. The four choices of the zones represent 1, 4, 16, and 64 depletion zones per fuel assembly. There are several reasons why such a variety of modeling was employed.

One aspect of considerable importance in generating accurate results for reactor-history calculations is maintaining the core-power level (or inferring equivalent full-power days). Generally, this was achieved in the calculations rather well with accounts kept of the cumulative average power. The anticipated change is then based on past history, allowing accurate compensation. Typically short-exposure periods (<100 days) were divided into three equal subintervals (occasionally five), while long ones (>200 days) were divided into eight. A relatively coarse accounting of the product of power and time is, however, used; the start- and end-power levels of each subinterval were averaged. Thus, a cycle done in four intervals with three substeps each produced 13 values of the power to be averaged. (The use of only a few intervals and substeps would cause the estimate of the average power level over an interval to be inaccurate.) Note that renormalization of the flux level to effect the desired power level was also done for the model applying redepletion.

Convergence criteria were selected representative of application (not tight for high-quality benchmarking) to produce results typical of code use. The maximum relative flux change over a full outer iteration was limited to <0.0005. This does not lead to tightly converged results as for the power densities, but they should be adequate recognizing there is some deviation from precision. Elaborate inhouse checking was desired to avoid errors in modeling, somewhat justifying the use of two different exposure models. The availability of results from the FRG problem originators was helpful. The simpler models are easy to describe, but an elaborate description and complexity add unreliability regarding accuracy of the modeling. The more variety in the modeling, the more accurate the determination and elimination of discretization errors. The use of two schemes for treating time should enhance the reliability of the results. Finally, computational costs had to be kept low promoting the use of a coarse grid of meshpoints and error evaluation.

Results

Only selected results are reported. Justification could not be found for the additional effort that would be necessary to extend the data processing for reporting elaborate information.

It may be noted that there is a certain consistency in the results due to the application of a set procedure of calculation. Possible variation in the results associated with levels of convergence, etc., is not indicated directly. Calculations not reported, however, indicated that only modest variation from the reported results is likely. Possibly the use of a consistent procedure produces accurate information about the difference in the results associated with a change in one or more of the variables. Any significant discrepancy in establishing the first-cycle time would likely directly affect the estimate of the second-cycle time. It is noted, however, that the first-cycle time seems to be somewhat more sensitive to the modeling variables than is the second-cycle time.

Primary information about calculations for two cycles is shown in Table 1. The dependence of cycle time on the modeling is shown. With the coarsest mesh the cycle times are grossly overestimated as is power-density peaking. Increasing the number of exposure (depletion) zones from 31 to 397 decreases the first- and second-cycle times by about 6 and 0.6 days, respectively; however, the estimated reactivity worth of the control-rod insertion at the end of the first cycle is reduced by only 0.03%. Calculation times depend directly on the number of meshpoints affecting the size of the neutronics problems and have only a small dependence on the number of zones and the size of the exposure problem. For case 102A, an independent procedure from the use of the fuel management code was used for refueling as a check.

Table 2 presents peak-power density data after refueling by fuel assembly, for start of cycle two. The modeling requirements to reduce the discretization error in these results are of considerable interest. Table 3 presents final average core-atom densities.

Gross results obtained with redepletion are shown in Table 4. The meaning of 2(3,3) is that both the first and the second cycle were divided into three subintervals and each of these intervals was treated with a first pass and then redepleted. With 2(1,1), there were three neutronics problems solved each cycle (plus one more for a 2-day exposure at the end of the cycle). With 2(3,3) there were seven neutronics problems solved each cycle, plus one at the end of two days, for a total of 16. [Note that with marchout (4,3) involves $5 + 4 = 9$ while (8,6) involves 16 neutronics problems. It may be noted that the coarsest case 2(1,1) is quite unstable - treating the first cycle in only one step results in severe oscillation of the final conditions that was not found to dampen with further redepletion calculations of the type done. Indeed, the behavior of the soluble boron prompted this writer to divide the first cycle into uneven periods (like 14 and then 34) for a 2(2,2) calculation. This proved to be quite unstable also. Stability was effected only by reducing the redepletion step time to under 200 days.] Cases 262, 263, 266 show the effect of refining the procedure in regard to modeling time with increasing intervals.

Data of peak-power density by assembly are given in Table 5 for the redepletion cases, and the final atom densities are shown in Table 6.

Information about the soluble boron requirements is shown in Table 7 for several cases.

It should be noted that end-of-cycle conditions do depend on the exposure time. Thus, if the cycle time was underestimated, the fuel consumption would be low, etc. So the accuracy for the

specific modeling does depend on how well the end-of-cycle state is established. Not much discrepancy was found for the results reported.

The results were analyzed to extract simple parameters for extraction of the discretization error contributions. These were then used to predict hopefully more accurate results. Shown in Table 8 are results obtained for the straight marchout case 420 and the inferred answers obtained by extrapolation. One assumption made was that the discretized meshpoint arrangement error contribution is proportional to the square of the meshspacing (inversely proportional to the number of one-dimensional points squared). Assuming that an error vector type of discretization error contribution is applicable, the resulting equation that was used to process the straight marchout results is

$$\begin{aligned}
 x_{\infty} = x_{i,j,m} &+ \left(\frac{\lambda_i}{1-\lambda_i} \right) (x_{i,j,M} - x_{i-1,j,M}) \\
 &+ \frac{x_{i,j,M} - x_{i,j-1,M}}{\left[\left(\frac{J}{J-1} \right)^2 - 1 \right]} \\
 &+ \left(\frac{\gamma_m^4}{1-\gamma_m^4} \right) (x_{i,j,M} - x_{i,j,M/3}), \quad (1)
 \end{aligned}$$

where x is some result, subscript i refers to an index referencing time, j refers to an index representing meshpoints, and m refers to an index referencing exposure zones. Thus, each of the three discretization error contributions are measured separately and extracted individually,

$$\begin{aligned}
 \lambda_i &= \frac{x_{i,j,m} - x_{i-1,j,m}}{x_{i-1,j,m} - x_{i-2,j,m}}, \quad \lambda_i \approx 0.338; \\
 \gamma_m \left(\frac{1-\gamma_m^4}{1-\gamma_m} \right) &= \frac{x_{i,j,m} - x_{i,j,m/3}}{x_{i,j,m/3} - x_{i,j,m/6}}, \quad \gamma_m \approx 0.208;
 \end{aligned}$$

where for

$$\begin{aligned}
 \gamma \left(\frac{1-\gamma^4}{1-\gamma} \right) &= \alpha; \\
 \gamma &= \frac{\alpha}{1+\alpha-\gamma^4},
 \end{aligned}$$

which can be solved iteratively.

REFERENCES

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2. D. R. Vondy, T. B. Fowler, G. W. Cunningham, *The Bold Venture Computation System for Nuclear Reactor Core Analysis, Version III*, USDOE Report, ORNL-5711 (June 1981).

Table 1. Pressurized water reactor core depletion benchmark problem results (straight marchout calculations, quarter core, two-dimensional, mesh-centered finite difference)

Case	101	102	103	104	105	113	202	203	204	213	214	219	303	304	305	315	404	405	415	420
Exposure zones	31	31	31	31	31	31	102	102	102	102	102	102	397	397	397	397	1566	1566	1566	1566
Meshpoints	81	289	289	4624	18496	1156	289	1156	4624	1156	4624	4624	18496	18496	18496	18496	18496	18496	18496	18496
Points/exposure zone	1.16	4	4	64	256	4	4	4	64	4	64	16	16	4	16	16	4	16	16	16
Points/fuel assembly	1.16	4	4	64	256	16	4	16	64	16	64	64	16	64	256	256	64	256	256	256
Steps in time	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)	(8,6)	(4,3)	(4,3)	(4,3)	(8,6)	(8,6)	(12,9)	(4,3)	(4,3)	(4,3)	(8,6)	(4,3)	(4,3)	(8,6)	(12,9)
Cycle time (days)																				
First cycle	420.41	399.23	399.08	390.57	387.93	387.16	391.33	395.33	386.20	383.35	384.13	384.29	385.05	382.12	381.32	382.01	381.80	380.89	381.65	381.84
Second cycle	271.50	268.35	268.41	265.99	264.69	264.58	264.92	268.17	265.57	264.53	264.62	263.60	263.27	264.31	264.60	263.06	264.25	263.92	263.02	262.70
Peak power density (W/cm ³), core average 93 W/cm ³																				
Initial, rods in	32.95	189.17	—	296.53	327.48	315.54	296.53	188.25	296.53	328.09	328.09	328.09	296.53	327.48	334.89	334.89	327.45	336.12	336.12	336.12
First cycle, start	148.82	121.70	122.17	118.95	122.31	126.33	119.08	121.51	118.81	117.76	117.52	118.22	116.75	122.29	128.23	126.10	122.36	126.11	126.11	126.11
First cycle, end	103.22	112.00	112.10	115.46	118.98	121.98	115.72	109.40	112.90	116.08	116.31	116.67	111.87	114.74	117.66	117.51	113.64	115.85	116.18	116.29
Second cycle, start	229.22	186.20	185.32	165.19	155.91	152.95	165.27	181.18	159.11	149.56	159.11	149.63	149.47	147.72	144.08	145.08	147.42	144.60	144.72	144.60
Second cycle, end	128.86	124.35	124.46	125.03	125.32	125.18	125.30	124.39	124.83	125.00	125.06	125.24	124.03	124.30	124.66	124.52	124.27	124.74	124.36	124.49
Fuel loading (kg), initial 1.68746, 0.77200 fresh added on refueling (quarter core)																				
First cycle, end	1.14725	1.17510	1.17521	1.18692	1.19137	1.19137	1.18742	1.18023	1.19257	1.19634	1.19112	1.19698	1.19723	1.19398	1.19941	1.19979	1.19806	1.19969	1.19997	1.20019
Second cycle, start	1.38461	1.61234	1.61239	1.62321	1.62672	1.62675	1.62298	1.61653	1.62776	1.63082	1.62759	1.63073	1.63073	1.62888	1.63210	1.63443	1.63232	1.63325	1.63320	1.63317
Second cycle, end	1.22998	1.26553	1.26545	1.28126	1.28941	1.28732	1.28170	1.27022	1.28839	1.29146	1.28690	1.29203	1.29210	1.28779	1.29298	1.29506	1.29333	1.29492	1.29540	1.29555
Initial soluble boron (ppm)																				
Rods in, no Xe	1021.2	629.5	—	554.1	537.5	532.0	554.1	629.5	554.1	537.6	554.1	537.6	554.1	537.5	531.9	531.9	537.5	532.2	532.2	532.2
Rods out, no Xe	1419.7	1312.9	—	1287.6	1280.7	1278.6	1287.6	1312.8	1287.6	1280.7	1280.7	1280.7	1287.6	1280.7	1278.6	1278.6	1280.8	1278.6	1278.6	1278.6
Rods out, with Xe	1155.8	1070.5	1070.5	1038.9	1032.6	1030.6	1048.6	1067.9	1048.0	1041.6	1041.6	1041.6	1047.8	1041.4	1037.9	1039.4	1041.4	1039.4	1039.4	1039.4
Conditions at end of first cycle, before refueling, no Xe																				
Soluble boron (ppm)	248.2	247.8	—	248.1	247.3	247.4	247.8	247.9	247.6	247.5	247.9	248.1	247.7	247.7	255.0	247.9	247.7	247.7	248.0	248.2
k, rods in	0.94903	0.93497	—	0.92891	0.92674	0.92597	0.92872	0.93474	0.92861	0.92645	0.92640	0.92638	0.92851	0.92637	0.92563	0.92562	0.92637	0.92563	0.92561	0.92559
Conditions after refueling, soluble boron (ppm)																				
Rods in, no Xe	1268.4	780.4	—	598.8	540.7	522.3	598.0	783.6	604.2	548.5	603.3	547.4	546.7	550.5	536.4	532.8	551.0	534.6	533.6	533.3
Rods out, no Xe	1560.7	1325.9	—	1268.0	1253.8	1250.1	1266.4	1331.2	1275.7	1265.5	1274.6	1262.3	1278.0	1266.2	1267.8	1262.1	1267.0	1264.1	1262.9	1262.8
Rods out, with Xe	1232.1	1025.9	1025.8	979.4	968.2	965.2	977.8	1031.3	987.3	977.8	986.0	976.5	976.1	989.4	982.3	976.8	980.9	978.8	977.6	977.1
Computer time (min)	1.2	1.8	1.4	3.8	14.5	69.5	5.4	2.1	4.1	12.8	5.7	16.9	20.4	13.7	69.3	99.8	16.2	73.0	99.1	124.4

Table 2. Peak power density by fuel assembly after refueling^a (W/cm³)

Case	102	103	105	203	204	219	224	303	304	305	315	404	405	415	420
Exposure zones	31	31	31	102	102	102	102	397	397	397	397	1566	1566	1566	1566
Meshpoints	289	1156	18496	4624	4624	4624	4624	1156	4624	18496	18496	4624	18496	18496	18496
Time steps	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)	(8,6)	(12,9)	(4,3)	(4,3)	(4,3)	(8,6)	(4,3)	(4,3)	(8,6)	(12,9)
Assembly ^b [x,y](x',y')															
[1,1]	32.286	50.045	59.238	56.114	63.577	63.743	63.783	57.615	65.966	68.725	68.754	66.189	69.259	68.910	69.095
[2,1]	46.382	70.646	86.317	76.969	88.344	88.364	88.368	78.899	91.531	96.649	96.514	92.600	97.461	97.178	97.378
[3,1]	42.642	61.854	71.153	65.575	73.414	73.554	73.619	66.461	74.998	77.880	77.934	74.870	78.072	78.023	78.172
[4,1]	50.243	66.792	73.495	70.509	75.956	75.863	75.888	71.272	77.020	78.595	78.488	76.977	78.883	78.725	78.735
[5,1]	77.941	95.593	106.231	94.263	100.552	100.442	100.464	94.357	101.863	104.202	103.997	102.693	104.880	104.648	104.641
[6,1]	77.299	85.363	90.498	84.722	88.152	88.013	88.035	84.567	88.024	89.561	89.439	87.750	89.281	89.203	89.211
[7,1]	100.372	102.066	106.566	107.238	106.966	106.964	107.033	109.759	109.367	109.590	109.550	110.481	110.729	110.751	110.847
[8,1]	127.804	117.727	112.700	119.016	115.250	115.258	115.336	119.493	115.808	114.463	114.435	115.988	114.639	114.684	114.794
[2,2]	44.316	65.771	76.334	68.727	76.986	77.044	77.079	69.337	78.847	82.335	82.232	78.016	81.829	81.615	81.781
[3,2]	63.399	88.977	103.301	90.963	101.496	101.416	101.428	90.588	101.842	105.596	105.424	101.877	105.965	105.716	105.791
[4,2]	69.498	91.092	102.990	92.361	100.636	100.456	100.465	93.198	101.273	104.395	104.144	102.158	105.330	105.058	105.036
[5,2]	67.927	81.503	88.908	81.875	87.021	86.924	86.956	81.931	87.110	89.043	88.965	86.886	88.809	88.768	88.769
[6,2]	125.417	132.644	135.274	131.935	133.958	133.859	133.921	131.382	133.799	134.149	134.049	133.735	134.134	134.084	134.092
[7,2]	80.465	84.010	87.759	83.528	85.820	85.760	85.797	83.440	85.674	86.798	86.905	85.182	86.508	86.450	86.549
[8,2]	125.326	114.297	109.791	114.644	111.219	111.215	111.282	114.813	111.407	110.422	110.387	111.477	110.513	110.548	110.646
[3,3]	49.302	66.692	75.515	69.488	76.927	77.044	77.121	70.283	78.248	80.588	80.669	78.130	80.777	80.824	80.873
[4,3]	76.544	97.637	107.988	103.639	112.541	112.486	112.533	103.811	114.086	117.255	117.164	113.470	116.676	116.546	116.545
[5,3]	81.525	94.738	104.082	98.413	105.236	105.193	105.243	100.865	108.206	110.456	110.376	108.899	111.310	111.119	111.144
[6,3]	78.752	82.636	87.503	82.246	85.571	85.540	85.572	82.117	85.376	86.910	86.905	85.124	86.684	86.731	86.732
[7,3]	92.433	89.811	90.246	88.061	87.719	87.771	87.764	87.623	87.061	87.383	87.472	86.746	87.026	87.152	87.157
[8,3]	133.083	119.820	115.108	117.368	113.242	113.319	113.318	116.849	112.439	111.560	111.633	112.313	111.430	111.544	111.564
[4,4]	60.250	71.042	75.967	70.096	73.791	73.956	74.039	70.088	74.754	76.408	76.558	75.108	76.847	76.958	76.983
[5,4]	83.494	89.218	92.143	86.877	88.527	88.509	88.459	86.253	87.819	88.826	88.879	87.467	88.445	88.525	88.461
[6,4]	115.911	116.258	118.311	112.471	112.599	112.634	112.567	111.432	111.357	112.721	112.545	110.706	112.363	112.501	112.450
[7,4]	173.970	152.800	143.203	148.962	141.633	141.733	141.677	148.158	140.397	137.903	138.042	139.833	137.703	137.869	137.827
[8,4]	137.543	123.674	119.081	120.734	116.627	116.731	116.723	120.114	115.684	114.891	115.003	115.524	114.714	114.865	114.882
[5,5]	168.452	155.682	149.843	150.912	146.372	146.368	146.250	149.933	144.887	143.315	143.398	144.637	143.081	143.077	143.027
[6,5]	114.900	109.713	106.656	105.937	103.536	103.516	103.402	104.941	102.073	101.142	101.227	101.586	100.622	100.717	100.631
[7,5]	173.242	149.808	141.198	145.125	138.616	138.697	138.612	144.181	137.226	135.156	135.292	137.004	134.919	135.069	135.007
[8,5]	186.200	165.194	152.946	159.074	149.564	149.625	149.468	157.785	147.721	144.934	145.077	147.419	144.601	144.723	144.597
[6,6]	163.986	145.886	136.918	140.161	133.330	133.420	133.287	138.969	131.627	129.465	129.611	131.352	129.155	129.283	129.187
[7,6]															

^aStart of cycle 2, equilibrium xenon, critical soluble boron, core average 93 W/cm³.^bLocations of assemblies shown after refueling and repositioning []; original positions if moved shown ().

Table 3. Final core average nuclide atom densities in fuel assemblies (core contents end of cycle 2)
[Atoms/barr-cm $\times 10^6$]

Case	102	103	105	203	204	213	214	219	303	304	305	315	404	405	415	420
Zones	31	31	31	102	102	102	102	102	397	397	397	397	1566	1566	1566	1566
Meshpoints	289	1156	18496	1156	4624	1156	4624	4624	1156	4624	18496	18496	4624	18496	18496	18496
Steps in time	(4.3)	(4.3)	(4.3)	(4.3)	(4.3)	(8.6)	(8.6)	(12.9)	(4.3)	(4.3)	(4.3)	(8.6)	(4.3)	(4.3)	(8.6)	(12.9)
Nuclide																
^{235}U	93.605	95.782	96.622	96.499	97.238	96.650	97.379	97.389	96.730	97.455	97.573	97.820	97.516	97.633	97.872	97.921
^{238}U	17.950	17.557	17.401	17.431	17.296	17.413	17.280	17.280	17.389	17.257	17.233	17.228	17.243	17.220	17.188	17.182
^{239}Pu	6164.0	6165.8	6166.4	6166.2	6166.8	6166.3	6166.9	6166.8	6166.4	6167.0	6166.9	6167.0	6166.2	6166.2	6166.3	6166.3
^{240}Pu	27.327	26.870	26.671	26.709	26.542	26.619	26.455	26.426	26.655	26.489	26.449	26.350	26.472	26.433	26.335	26.302
^{241}Pu	7.4783	7.2902	7.2142	7.2369	7.1734	7.2263	7.1636	7.1612	7.2206	7.1588	7.1471	7.1297	7.1537	7.1425	7.1243	7.1205
^{242}Pu	5.5267	5.3829	5.3313	5.3483	5.3022	5.3530	5.3070	5.3109	5.3377	5.2933	5.2895	5.2845	5.2906	5.2866	5.2824	5.2823
^{243}Pu	1.3736	1.3605	1.3582	1.3531	1.3482	1.3605	1.3551	1.3588	1.3508	1.3463	1.3490	1.3516	1.3454	1.3479	1.3510	1.3526
BPa^a	0.18395	0.18364	0.18202	0.18769	0.18740	0.18685	0.18676	0.18630	0.18885	0.18860	0.18766	0.18778	0.18896	0.18804	0.18814	0.18807
^{135}Xe	2.1997-3	2.2006-3	2.2001-3	2.1959-3	2.1951-3	2.1930-3	2.1921-3	2.1905-3	2.1936-3	2.1926-3	2.1919-3	2.1892-3	2.1920-3	2.1912-3	2.1884-3	2.1874-3
FP^b	131.12	128.49	127.51	127.61	126.71	127.54	126.65	126.69	127.32	126.45	126.37	126.12	126.37	126.28	126.06	126.02

^aResidue burnable poison.

^bLumped fission products.

Table 4. Results obtained by redepletion for the pressurized water reactor core depletion benchmark problem

Case	151	162	261	262	263	264	266	362	363	373	383	473	483
Exposure zones	31	31	102	102	102	102	102	397	397	397	397	1566	1566
Meshpoints	289	1156	1156	1156	1156	1156	1156	1156	1156	4624	18496	4624	18496
Points/exposure zone	16	16	4	4	4	4	4	1	1	4	16	1	4
Points/fuel assembly	16	16	6	16	16	16	16	16	16	16	256	64	256
Exposure substeps	8	8	4	8	8	8	8	8	8	8	8	8	8
Steps in time	2(1,1)	2(2,2)	2(2,2)	2(2,2)	2(3,3)	2(4,4)	2(6,6)	2(2,2)	2(3,3)	2(3,3)	2(3,3)	2(3,3)	2(3,3)
Cycle time (days)													
First cycle	391.75	390.44	386.04	386.04	386.66	386.94	387.20	384.80	385.49	382.61	381.69	382.23	381.29
Second cycle	319.68	263.64	263.26	263.26	263.19	263.44	263.57	263.18	263.00	261.74	261.63	261.79	261.53
Peak power density (W/cm ³), core average 93 W/cm ³													
First cycle, start	122.17	119.34	119.08	119.08	119.08	119.08	119.08	119.07	119.07	122.45	126.43	122.44	126.18
First cycle, end	128.13	113.50	113.30	113.17	111.38	112.66	112.90	112.17	110.69	113.61	116.52	112.62	115.10
Second cycle, start	176.66	170.07	163.33	163.32	160.71	159.93	159.81	161.74	159.14	149.86	146.85	149.09	146.09
Second cycle, end	180.31	137.32	133.90	133.90	124.73	124.89	125.08	133.54	123.99	124.45	124.59	124.36	124.39
Flissile loading (kg), Initial 168746; 0.77200 fresh added on refueling (quarter core)													
First cycle, end	1.17348	1.18988	1.19006	1.19006	1.19215	1.19286	1.19327	1.19165	1.19362	1.19759	1.19878	1.19770	1.19903
Second cycle, start	1.61518	1.61987	1.62485	1.62486	1.62646	1.62696	1.62723	1.62621	1.62769	1.63084	1.63186	1.63109	1.63208
Second cycle, end	1.22237	1.28036	1.28622	1.28623	1.28715	1.28720	1.28723	1.28794	1.28871	1.29368	1.29519	1.29402	1.29552
Initial soluble boron with Xe (ppm)													
First cycle	1070.5	1048.6	1048.0	1048.0	1048.0	1048.0	1048.0	1047.8	1047.8	1041.4	1039.4	1041.3	1039.4
Second cycle	1025.3	972.4	980.9	980.9	983.4	984.2	984.5	983.0	985.6	976.0	973.9	976.7	974.5
Computer time (min)	0.95	2.8	3.0	3.1	4.0	5.0	6.8	4.8	4.4	13.4	41.1	18.8	63.6

Table 5. Peak power density by fuel assembly after refueling for cases using redepletion^a (W/cm³)

Case	162	262	263	264	266	363	373	383	473	483
Depletion zones	31	102	102	102	102	397	397	397	1566	1566
Meshpoints	1156	1156	1156	1156	1156	1156	4624	18496	4624	18496
Time steps	2(2,2)	2(2,2)	2(3,3)	2(4,4)	2(6,6)	2(3,3)	2(3,3)	2(3,3)	2(3,3)	2(3,3)
Assembly [x,y], (x',y')										
[1,1]	—	47.464	53.854	55.337	55.820	55.937	57.091	64.922	68.125	65.824
[2,1]	(8,1)	65.519	73.249	75.553	76.280	76.432	77.860	89.734	95.416	91.263
[3,1]	(7,1)	59.083	63.230	64.781	65.266	65.369	65.841	73.807	77.091	73.989
[4,1]	(6,1)	65.004	68.955	69.805	70.067	70.098	70.694	76.193	77.911	76.199
[5,1]	(8,2)	94.738	92.748	93.562	93.819	93.845	93.624	100.825	103.294	101.462
[6,1]	(2,1)	85.868	85.138	84.646	84.507	84.449	84.503	87.812	89.253	87.640
[7,1]	(8,2)	102.473	107.506	107.328	107.269	107.223	109.812	109.339	109.315	110.497
[8,1]	—	118.689	119.729	119.297	119.156	119.077	119.722	115.929	114.299	116.147
[2,2]	(6,6)	61.849	65.415	67.544	68.205	68.374	68.417	77.204	81.192	76.957
[3,2]	(7,6)	84.752	86.893	89.337	90.096	90.267	89.228	99.905	104.176	100.320
[4,2]	(8,3)	88.316	89.911	91.177	91.610	91.700	92.280	100.027	103.357	101.161
[5,2]	(5,2)	80.991	81.466	81.578	81.612	81.590	81.672	86.749	88.669	86.546
[6,2]	—	132.287	132.116	131.882	131.805	131.736	131.692	133.559	133.783	133.333
[7,2]	(5,4)	84.214	83.719	83.510	83.436	83.385	83.401	85.590	86.567	85.021
[8,2]	—	115.326	115.418	114.945	114.788	114.705	115.056	111.575	110.306	111.668
[3,3]	(5,5)	64.504	67.397	68.736	69.216	69.354	69.652	77.287	79.940	77.669
[4,3]	(8,4)	94.981	96.448	102.545	103.108	103.255	102.798	112.597	116.186	112.248
[5,3]	(7,5)	93.641	100.723	97.710	98.094	98.203	100.126	107.169	109.642	108.073
[6,3]	(6,3)	83.190	82.286	82.133	82.155	82.142	81.995	85.250	86.693	85.054
[7,3]	(7,2)	90.788	88.918	88.431	88.272	88.234	87.908	87.585	87.801	87.487
[8,3]	—	121.687	118.940	118.073	117.796	117.725	117.418	113.302	112.233	113.013
[4,4]	—	71.463	70.481	70.386	70.359	70.379	70.325	74.283	76.123	74.749
[5,4]	(4,3)	90.634	88.113	87.287	87.034	86.992	86.585	88.451	89.424	87.985
[6,4]	(7,4)	118.075	114.120	113.160	112.857	112.814	111.980	112.248	113.526	111.632
[7,4]	—	155.640	147.508	150.038	149.601	149.523	148.045	141.797	139.075	141.308
[8,4]	—	125.762	122.505	121.536	121.230	121.161	120.767	116.686	115.711	116.334
[5,5]	—	159.053	153.913	152.005	151.432	151.340	150.827	146.459	144.680	145.873
[6,5]	(3,2)	112.932	108.757	106.854	106.295	106.208	105.714	103.408	102.331	102.655
[7,5]	—	153.234	148.127	146.332	145.784	145.690	145.170	138.794	136.517	138.238
[6,6]	—	170.067	163.321	160.712	159.931	159.812	159.144	149.865	146.846	149.092
[7,6]	—	150.155	143.865	141.649	140.984	140.884	140.205	133.595	131.225	132.891

^aStart of cycle 2, equilibrium xenon, critical soluble boron, average 93 W/cm³.

Table 6. Final core average nuclide atom densities in fuel assemblies obtained by redepletion (core constant end of cycle 2)
[atom/barn-cm $\times 10^6$]

Case	151	162	261	262	263	266	362	363	373	383	473	483
Zones	31	31	102	102	102	102	397	397	397	397	1566	1566
Meshpoints	289	1156	1156	1156	1156	1156	1156	1156	4624	18496	4624	18496
Steps in time	2(1,1)	2(2,2)	2(2,2)	2(2,2)	2(3,3)	2(6,6)	2(2,2)	2(3,3)	2(3,3)	2(3,3)	2(3,3)	2(3,3)
Nuclide												
²³⁵ U	89.356	96.064	96.943	96.943	96.863	96.825	96.914	97.085	97.788	98.023	97.855	98.031
²³⁶ U	18.443	17.493	17.340	17.340	17.371	17.387	17.341	17.331	17.203	17.160	17.189	17.156
²³⁸ U	6155.8	6166.1	6166.8	6166.8	6166.6	6166.3	6166.5	6166.6	6167.2	6167.4	6166.6	6166.7
²³⁹ Pu	26.768	26.526	26.351	26.351	26.453	26.498	26.328	26.400	26.246	26.195	26.229	26.187
²⁴⁰ Pu	7.5002	7.2083	7.1448	7.1448	7.1806	7.2007	7.1475	7.1650	7.1059	7.0863	7.1005	7.0853
²⁴¹ Pu	6.0087	5.3756	5.3244	5.3243	5.3338	5.3454	5.3371	5.3239	5.2784	5.2637	5.2745	5.2645
²⁴² Pu	1.8372	1.3826	1.3679	1.3680	1.3629	1.3644	1.3757	1.36081	1.3539	1.3516	1.3525	1.3520
BP ^a	0.11696	0.17409	0.17952	0.17951	0.18437	0.18601	0.17899	0.18553	0.18622	0.18618	0.18672	0.18649
¹³⁵ Xe	2.0385-3	2.1890-3	2.1867-3	2.1867-3	2.1888-3	2.1880-3	2.1844-3	2.1871-3	2.1852-3	2.1849-3	2.1846-3	2.1837-3
FP ^b	142.31	128.30	127.14	127.40	127.24	127.36	127.30	126.97	126.09	125.80	125.99	125.79

^aResidue burnable poison.

^bLumped fission products.

Table 8. Results inferred from discretization error interpretation using marchout data

History for two cycles			Peak power density (W/cm ³)			Final core average Nuclide density (atom/barn-cm × 10 ⁶)		
Result	Calculated ^a	Extrapolated ^b	Zone	Calculated	Extrapolated	Nuclide	Calculated	Extrapolated
Cycle time (days)								
First cycle	381.84	381.60	[1,1]	69.095	70.18	²³⁵ U	97.921	97.98
Second cycle	262.70	262.48	[2,1]	97.378	99.07	²³⁶ U	17.182	17.172
Peak Power Density (W/cm³)			[3,1]	78.172	79.29	²³⁸ U	6166.3	6166.3
Initial, rods in	336.12	339.01	[4,1]	78.735	79.37	²³⁹ Pu	26.302	26.278
First cycle, start	126.11	127.36	[5,1]	104.641	105.37	²⁴⁰ Pu	7.1205	7.115
First cycle, end	116.29	117.06	[6,1]	89.211	89.72	²⁴¹ Pu	5.2823	5.281
Second cycle, start	144.60	143.62	[7,1]	110.847	110.96	²⁴² Pu	1.3526	1.3540
Second cycle, end	124.49	124.69	[8,1]	114.794	114.38	BP	0.18807	0.18774
Fissile loading (kg, quarter core)			[2,2]	81.781	83.11	¹³⁵ Xe	2.1874-3	2.1868-3
First cycle, end	1.20019	1.20081	[3,2]	105.791	107.18	FP	126.02	125.98
Second cycle, start	1.63317	1.63347	[4,2]	105.036	106.09			
Second cycle, end	1.29555	1.29613	[5,2]	88.769	89.41			
Initial soluble boron (ppm)			[6,2]	134.092	134.23			
Rods in, no Xe	532.2	530.4	[7,2]	86.549	87.02			
Rods out, no Xe	1278.6	1277.9	[8,2]	110.646	110.36			
Rods out, with Xe	1039.4	1038.7	[3,3]	80.873	81.77			
End of first cycle before refueling			[4,3]	116.545	117.61			
Soluble boron (ppm)	248.2	248.3	[5,3]	111.144	111.96			
k, rods in	0.92559	0.92534	[6,3]	86.732	87.25			
After refueling, soluble boron (ppm)			[7,3]	87.157	87.25			
Rods in, no Xe	533.3	527.7	[8,3]	111.564	111.28			
Rods out, no Xe	1262.8	1261.8	[4,4]	76.983	77.57			
Rods out, with Xe	977.1	976.2	[5,4]	88.461	88.74			
			[6,4]	112.450	112.99			
			[7,4]	137.827	137.10			
			[8,4]	114.882	114.62			
			[5,5]	143.027	142.46			
			[6,5]	100.631	100.28			
			[7,5]	135.007	134.29			
			[6,6]	144.597	143.61			
			[7,6]	129.187	128.42			

^aCase 420.^bCase data used: time 415→420→; mesh 404→405→; zones 315→415→.

Reviewer's Notes

I have summarized the relevant points of comparison between the two solutions in the following Tables and Figure. Because the solution techniques and the type of reported results were significantly different, all results are not directly comparable. I have selected those results which are common to both solutions. In general, they are "integral" measures of the solution (e.g., core average nuclide density). Even though the details may be different, the points of commonality between the two solutions support the "goodness" of each solution.

As shown in Tables 1-4, the agreement (generally far better than 1%) between the values for critical boron concentrations, equivalent full power days, core average nuclide densities at the end of cycle 2, and control rod worths calculated by the two solutions is remarkable. Figure 1 shows that the maximum assembly power distribution at BOC2 and xenon equilibrium is calculated to be virtually identical between the two solutions.

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Table 1. Initial Critical Boron Concentration (ppm)

<u>Time</u>	<u>KWU</u>	<u>ORNL 1</u>	<u>ORNL 2</u>	<u>ORNL 3</u>	<u>% Difference</u>
BOC1 w/o Xe	1277.8	1278.6	-	1277.9	-0.008
BOC1 eq. Xe	1038.6	1039.4	1039.4	1038.7	-0.010
BOC2 w/o Xe	1260.6	1262.8	-	1261.8	-0.095
BOC2 eq. Xe	979.5	977.1	974.5	976.2	0.337

Table 2. Equivalent Full Power Days per Cycle

<u>Time</u>	<u>KWU</u>	<u>ORNL 1</u>	<u>ORNL 2</u>	<u>ORNL 3</u>	<u>% Difference</u>
EOC1	381.49	381.84	381.29	381.60	-0.029
EOC2	262.04	262.07	261.53	262.48	-0.168

Note: ORNL1 = ORNL straight marchout

ORNL2 = ORNL redepletion

ORNL3 = ORNL extrepolated

% difference = $(KWU - ORNL3 / KWU \times 100)$

Table 3. Core Average Nuclide Densities ($10^{24}/\text{cm}^3$) at EOC2

<u>Nuclide</u>	<u>KWU</u>	<u>ORNL1</u>	<u>ORNL2</u>	<u>ORNL3</u>	<u>% Diffusion</u>
U235	.98062E-4	.97921	.98031	.9798	0.083
U236	.17164E-4	.17182	.17156	.17172	-0.047
U238	.61675E-2	.61663	.61667	.61663	-0.019
PU239	.26234E-4	.26302	.26187	.26278	-0.167
PU240	.71044E-5	.71205	.70853	.7115	-0.149
PU241	.52732E-5	.52823	.52645	.5281	-0.148
PU242	.13521E-5	.13526	.13520	.13540	-0.141
BP	.18791E-6	.18807	.18649	.18774	-0.090
XE135	.21839E-8	.21874	.21837	.21868	-0.133
FP	.12583E-3	.12602	.12579	.12598	-0.119

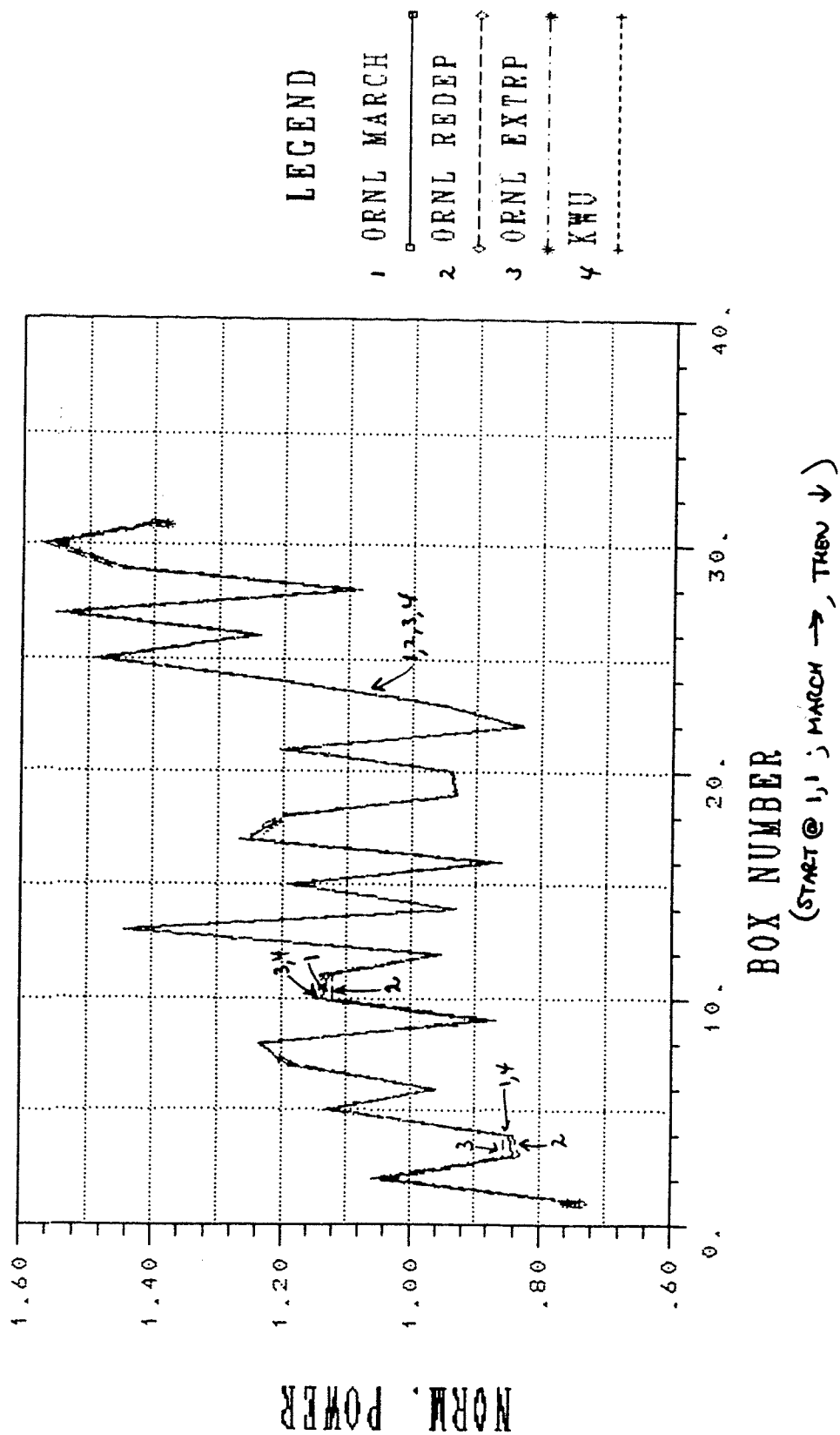
Table 4. Control Rod Worths

<u>Time</u>	<u>CO (ppm) unradded</u>			<u>CR (ppm) radded</u>		
	<u>KWU</u>	<u>ORNL</u>	<u>% Difference</u>	<u>KWU</u>	<u>ORNL1</u>	<u>% Difference</u>
BOC1	1277.8	1278.6	-0.063	530.1	532.2	-0.396
BOC2	1260.0	1268.2	-0.651	526.4	533.3	-1.311
EOC1	248.2	248.2	0	248.2	248.2	0

Control Rod Worths

	<u>KWU</u>	<u>ORNL1</u>	<u>% Difference</u>
BOC1 (ppm)	747.7	746.4	0.174
BOC2 (ppm)	734.2	734.9	-0.095
EOC1 (% $\Delta\rho$)	-8.078	-8.039	0.483

FIG 1. MAXIMUM ASSEMBLY POWER
(BOC 2, equil. xenon)



Results From Page 897

Cycle 1

Time (EFPD)	Burnup (MWd/kgU)	Boron (ppm)	
0	0	1277.8	(without Xe)
0	0	1038.6	(with Xe)
30	1.098	1081.7	
60	2.197	1069.2	
90	3.295	1019.0	
120	4.394	944.1	
150	5.492	853.1	
180	6.590	751.5	
210	7.689	643.5	
240	8.787	531.7	
270	9.886	418.1	
300	10.984	304.3	
330	12.082	191.0	
360	13.181	79.1	
381.49	13.968	0.0	

Cycle 2

Time (EFPD)	Burnup (MWd/kgU)	Boron (ppm)	
0	8.754	1260.6	(without Xe)
0	8.754	979.5	(with Xe)
30	9.853	891.5	
60	10.951	789	
90	12.05	677.5	
120	13.148	560.9	
150	14.246	442	
180	15.345	322.5	
210	16.443	203.4	
240	17.542	85.5	
262.04	18.349	0	