ANL-7416 Supplement 2
Mathematics and Computers
(UC-32)

ARGONNE CODE CENTER:
BENCHMARK PROBLEM BOOK

Prepared by the
Computational Benchmark Problems Committee of the
MATHEMATICS AND COMPUTATION DIVISION
OF THE AMERICAN NUCLEAR SOCIETY

Revised June 1977

Benchmark Problems Included

11. Multi-dimensional (x-y-z) LWR Model
13. Neutron Transport in a BWR Rod Bundle
14. Multi-dimensional (x-y-z) BWR Model
15. Neutronic Depletion Benchmark Problems

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ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

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BENCHMARK SOURCE SITUATION

Identification: 15

Date Submitted: July 1976 By: M. V. Gregory (SRL)

Date Adopted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
R. R. Lee (CE)

Descriptive Title

Neutronic Depletion Benchmark Problems

Suggested Function

Test solutions of the neutronic depletion equations.

Configuration

An infinite homogeneous medium with the isotopic concentrations given below. At time zero, the neutron flux becomes nonzero.

Details

Isotope	Concentration, atom/(barn-cm)
^{235}U	0.74003E-4*
^{238}U	0.69360E-2
All others	0

* E-X = 10^{-X}

BENCHMARK PROBLEM

Identification: 15-A1 Source Situation ID.15

Date Submitted: July 1976 By: M. V. Gregory (SRL)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
R. R. Lee (CE)

Descriptive Title

Two-Group Constant Parameter Point Depletion Problem

Reduction from Source Situation

1. Solution of isotopic depletion equations at a point with constant flux and cross sections.

$$\frac{d}{dt}\underline{N}(t) = \underline{\underline{A}}\,\underline{N}(t)$$

where

$\underline{N}$ = vector of isotopic concentrations

$\underline{\underline{A}}$ = net production matrix coupling the isotopes

The general ijth entry in $\underline{\underline{A}}$ (i.e., the production rate of isotope i from isotope j) is

$$A_{ij} = Y_{ij}\sum_g \sigma^g_{fj}\phi^g + \lambda_{ij} + \sum_g \sigma^g_{c_{ij}}\phi^g$$

where

g = energy group index

Y_{ij} = fission yield of isotope i from the fissioning of isotope j (Y_{ii} is defined as -1.0)

σ^g_{fj} = microscopic fission cross section of isotope j in group g

ϕ^g = flux in group g

λ_{ij} = decay constant for production of isotope i from the decay of isotope j (λ_{ii} is the negative of the decay constant)

$\sigma^g_{c_{ij}}$ = microscopic capture cross section in group g for isotope j that produces i ($\sigma^g_{c_{ii}}$ is the negative of the capture cross section)

2. Constant two-group flux:

   Group 1 = 6.1374E+14 n/(cm^2-sec)

   Group 2 = 2.5078E+14 n/(cm^2-sec)

3. Fission product yields are defined in Table B.1.

4. Decay constants are defined in Table B.2.

5. Microscopic cross sections are defined in Table B.3.

6. Depletion chains are defined in Figure B.1 where the dashed line processes (α-, β^+-decay) are excluded, resulting in a triangular matrix for $\underline{\underline{A}}$.

Expected Primary Results

1. Variation of isotopic concentrations with time; 50-day concentrations.

2. Calculational statistics.

Solutions Available

1. Fourth-order Runge-Kutta integration of depletion equations, various time-step sizes: 15-A1-1

2. Analytical and finite-difference solutions for various time-step sizes: 15-A1-2, 15-A1-3

TABLE B.1. Fission Product Yield, %

Fission Product	Fissioning Isotope ^{235}U	^{238}U	^{239}Pu	^{241}Pu
^{135}I	6.17	5.78	6.93	6.26
^{135}Xe	0.24	0.22	0.27	0.24
^{149}Pm	1.13	2.10	1.30	1.20
^{147}Nd	2.36	2.80	2.05	2.20
Long-Lived Fission Products	90.10	89.10	89.45	90.10

TABLE B.2. Decay Constants

Isotope	Emitted Particle	Decay Constant, sec^{-1}
^{135}I	β^-	2.874E-5*
^{135}Xe	β^-	2.093E-5
^{147}Nd	β^-	7.228E-7
^{147}Pm	β^-	8.289E-9
^{148}Pm	β^-	1.488E-6
^{148m}Pm	β^-	1.976E-7
^{149}Pm	β^-	3.626E-6
^{237}U	β^-	1.190E-6
^{239}U	β^-	4.915E-4
^{238}Np	β^-	3.820E-6
^{239}Np	β^-	3.410E-6
^{240}Np	β^-	1.583E-3
^{241}Pu	β^-	1.680E-9
^{243}Pu	β^-	3.886E-5
^{242}Am	β^-	9.930E-6
^{244}Am	β^-	4.440E-4

* $E\text{-}X = 10^{-X}$

TABLE B.3. Microscopic Cross Sections, barns*

Isotope	σ_{c_1}	σ_{c_2}	σ_{f1}	σ_{f2}	$\sigma^{m}_{(n,\gamma)_1}$	$\sigma^{m}_{(n,\gamma)_2}$
^{234}U	33.575	26.368	0.42744	0	0	0
^{235}U	5.9872	26.420	12.370	148.18	0	0
^{236}U	16.859	1.4399	0.16664	0	0	0
^{237}U	16.991	132.12	0.17139	0.55512	0	0
^{238}U	0.53258	0.73141	0.081338	0	0	0
^{239}U	0.40015	6.1613	0.27283	4.2009	0	0
^{237}Np	24.072	71.864	0.41867	0.0094250	0	0
^{238}Np	5.2648	55.512	47.412	555.12	0	0
^{239}Np	26.341	16.654	0	0	0	0
^{240}Np	0	0	0	0	0	0
^{238}Pu	7.3125	119.91	1.5815	3.5496	0	0
^{239}Pu	9.8658	196.77	14.403	348.89	0	0
^{240}Pu	366.09	96.479	0.54033	0.016744	0	0
^{241}Pu	8.0305	152.24	29.986	352.73	0	0
^{242}Pu	51.820	5.1903	0.43460	0	0	0
^{243}Pu	11.030	21.649	28.382	49.960	0	0
^{241}Am	50.633	392.68	1.1130	2.3817	6.7486	39.543
^{242}Am	2.3381	0	31.137	693.90	0	0
^{242m}Am	20.016	444.09	108.79	1776.4	0	0
^{243}Am	91.056	24.080	0.30784	0	0	0
^{244}Am	0	0	26.192	403.29	0	0
^{242}Cm	3.1202	1.7185	0	0.83267	0	0
^{243}Cm	9.9059	69.389	92.299	194.29	0	0
^{244}Cm	32.129	3.6915	1.5663	0.33307	0	0
^{245}Cm	4.8993	82.972	37.165	537.15	0	0
^{135}I	0	0	0	0	0	0
^{135}Xe	243.47	1064780.0	0	0	0	0
^{147}Nd	0	0	0	0	0	0
^{147}Pm	248.78	65.814	0	0	117.44	31.087
^{148}Pm	3368.4	420.09	0	0	0	0
^{148m}Pm	2921.0	7561.6	0	0	0	0
^{149}Pm	0	0	0	0	0	0
^{149}Sm	105.85	23387.4	0	0	0	0
Fission Products	10.376	19.429	0	0	0	0

*

σ_{c_1} = capture in Group 1 [all captures except fission and (n,2n); includes (n,γ) to excited state, if any].

σ_{c_2} = capture in Group 2.

σ_{f_1} = fission in Group 1.

σ_{f_2} = fission in Group 2.

$\sigma^{m}(n,\gamma)_1$ = (n,γ) to first excited state, Group 1.

$\sigma^{m}(n,\gamma)_2$ = (n,γ) to first excited state, Group 2.

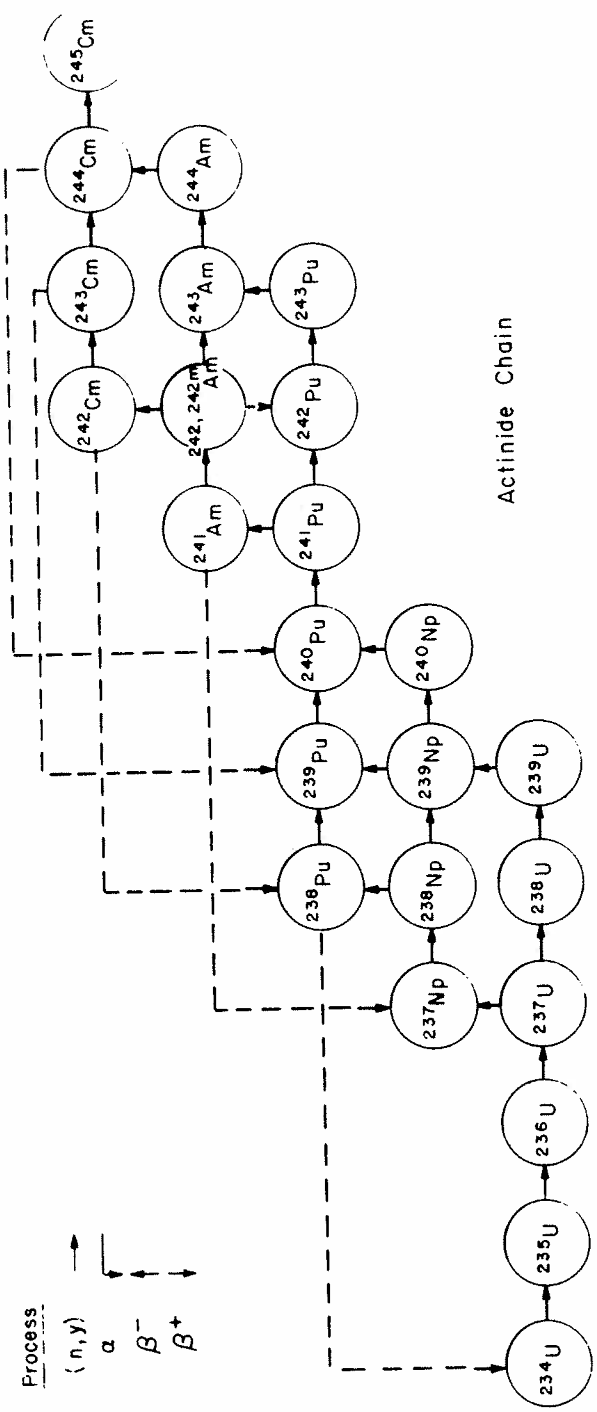
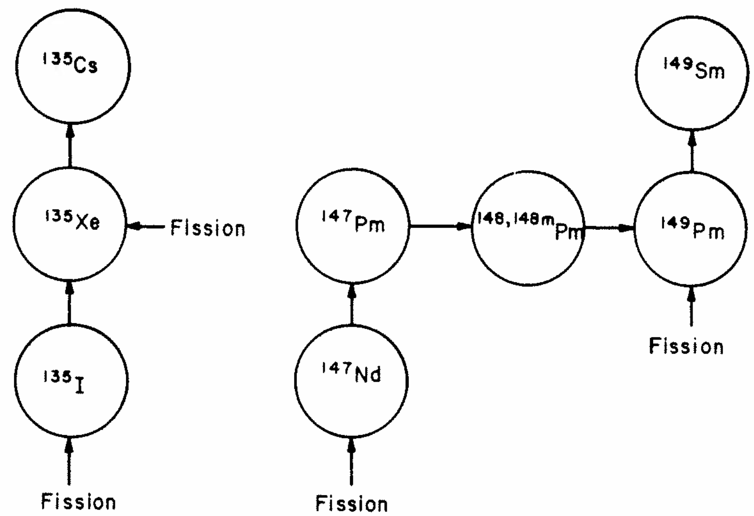


FIGURE B.1 Depletion Chains



Fission Product Chains

FIGURE B.1 Depletion Chains (Continued)

BENCHMARK PROBLEM SOLUTION

Identification: 15-A1-1 Benchmark Problem ID.15-A1

Date Submitted: July 1976 By: M. V. Gregory (SRL)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)

R. R. Lee (CE)

Descriptive Title

Two-Group Constant Parameter Point Depletion Problem

Technique Used

1. Fourth-order Runge-Kutta-Gill numerical integration technique (Reference 1).

2. Time-step selection may be automatic or user defined. Automatic time-step selection is defined by

$$\Delta t = 0.13/\left|\left(\sum_i A_{ij}\right)_{max}\right|$$

where the denominator is the maximum absolute columnar sum in the $\underline{\underline{A}}$ matrix. Some isotopes have very large time constants (e.g., decay very rapidly) and are excluded from the time-step determination and defined in equilibrium:

$$N_i(t + \Delta t) = \left(\sum_{j\neq i} A_{ij}N_j/A_{ii}N_i\right)_t \cdot N_i(t)$$

The criterion is that for $\left|\left(\sum_i A_{ij}\right)_{max}\right| > 1.0E-5$, the isotope j is defined to be in equilibrium. Solutions will be presented both with and without the use of defined equilibrium.

Computer

IBM 360/195

Program

BURGR

References

1. *The JOSHUA System, Volume 10.1 Generalized Reactor Analysis Subsystem: Physics.* (draft) DPSTM-500 (1975).

2. M. M. Anderson et al., "Three-Dimensional Coupled Neutronics and Engineering Calculation of Savannah River Reactors," CONF-750413, Vol. II., p. VI-123.

Primary Results

Isotopic concentrations at 50 days are summarized in Table C.1 for a range of time-step sizes. The "Δt = 1 min" and "Δt = 10 min" results make no assumptions about equilibrium. The "Δt = 1 hr" solution uses the standard equilibrium condition. The "$\Delta t \sim 6.09$ hr" solution uses automatic time-step selection and the standard equilibrium condition to set Δt to 21923 seconds.

Computational statistics are summarized at the end of the table.

TABLE C.1. SRL Solutions to Constant-Parameter Problem, atoms/(barn-cm) at 50 days

	Δt Integration Time Step		
Isotope	1 min, 10 min[a]	1 hour[b]	~6.09 hours[c]
^{235}U	0.583393E-4	0.583393E-4	0.583391E-4
^{236}U	0.286054E-5	0.286054E-5	0.286059E-5
^{237}U	0.206091E-7	0.206091E-7	0.206095E-7
^{238}U	0.691915E-2	0.691915E-2	0.691915E-2
^{239}U	0.718357E-8	0.718356E-8	0.718356E-8
^{237}Np	0.450818E-7	0.450818E-7	0.450837E-7
^{238}Np	0.324870E-9	0.324870E-9	0.324885E-9
^{239}Np	0.102943E-5	0.102943E-5	0.102943E-5
^{240}Np	0.132291E-10	0.132291E-10	0.132291E-10
^{238}Pu	0.147272E-8	0.147272E-8	0.147282E-8
^{239}Pu	0.105746E-4	0.105774E-4	0.105717E-4
^{240}Pu	0.995886E-6	0.996402E-6	0.995317E-6
^{241}Pu	0.334194E-6	0.334479E-6	0.333869E-6
^{242}Pu	0.163642E-7	0.163842E-7	0.163404E-7
^{243}Pu	0.136270E-10	0.139428E-10	0.139055E-10
^{241}Am	0.586414E-9	0.587114E-9	0.585583E-9
^{242}Am	0.617228E-11	0.669732E-11	0.667986E-11
^{242m}Am	0.504837E-11	0.505566E-11	0.503955E-11
^{243}Am	0.456826E-9	0.470206E-9	0.467301E-9
^{244}Am	0.637702E-13	0.658008E-13	0.653939E-13
^{242}Cm	0.549962E-10	0.610962E-10	0.607208E-10
^{243}Cm	0.115382E-12	0.130911E-12	0.129927E-12
^{244}Cm	0.206820E-10	0.214767E-10	0.212434E-10
^{245}Cm	0.243404E-12	0.254329E-12	0.251092E-12
^{135}I	0.882735E-8	0.883435E-8	0.883286E-8
^{135}Xe	0.914750E-9	0.915483E-9	0.915350E-9
^{147}Nd	0.121154E-6	0.121163E-6	0.121144E-6
^{147}Pm	0.201810E-6	0.201826E-6	0.201795E-6
^{148}Pm	0.457029E-8	0.457067E-8	0.456997E-8
^{148m}Pm	0.386726E-8	0.386758E-8	0.386699E-8
^{149}Pm	0.199678E-7	0.199694E-7	0.199662E-7
^{149}Sm	0.119776E-7	0.119786E-7	0.119766E-7
Fission Products	0.145224E-4	0.145238E-4	0.145212E-4
CPU time, sec IBM 360/195	429.8 (Δt = 1 min) 43.2 (Δt = 10 min)	7.0	1.2

a. No equilibrium assumption.

b. Equilibrium assumed.

c. Equilibrium assumed and time step automatically selected.

BENCHMARK PROBLEM SOLUTION

Identification: 15-A1-2 Benchmark Problem ID.15-A1

Date Submitted: July 1976 By: B. A. Weate (GA)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
R. R. Lee (CE)

Descriptive Title

Two-Group Constant Parameter Point Depletion Problem

Technique Used

1. In solving the nuclide depletion equation over a time-step, the flux and reaction rate cross sections are assumed constant.

2. Both analytic and difference solutions are defined. The difference solution is used when the nuclide and all chain nuclides above or below it have a removal rate $[(\lambda + \sigma\phi)\Delta t]$ less than a (where $e^{-a} - \frac{2-a}{2+a} = \varepsilon$ {error criterion}). Otherwise, the analytic solution is used. For the present solution, $a = 0.05$ and $\varepsilon = 10^{-5}$.

Computer

Univac 1110

Program

BURNUP

Reference

1. M. R. Wagner, *GAUGE, A Two-Dimensional Few Group Neutron Diffusion-Depletion Program for a Uniform Triangular Mesh.* GA-8307, "Chapter 4 - Nuclear Depletion Calculation" (1968).

Primary Results

Fifty-day isotopic concentrations are described in Table C.5 for time-step sizes ranging from 2 days to 10 minutes.

Computational statistics are defined in the table by SUP time, which is a combination of CPU time and I/Ø time.

TABLE C.5. GA Solutions to Constant Parameter Problem.
atoms/(barn-cm) at 50 days

Isotope	GA Small Time Step 10 min	1 hr	2 days
^{235}U	5.83386-5	5.83387-5	5.83393-5
^{236}U	2.86058-6	2.86058-6	2.86060-6
^{237}U	2.06103-8	2.06103-8	2.05937-8
^{238}U	6.91924-3	6.91938-3	6.91923-3
^{239}U	7.18270-9	7.18385-9	7.18404-9
^{237}Np	4.50845-8	4.50846-8	4.50933-8
^{238}Np	3.24895-10	3.24895-10	3.25098-10
^{239}Np	1.02945-6	1.02947-6	1.02946-6
^{240}Np	1.32294-11	1.32297-11	1.32295-11
^{238}Pu	1.47281-9	1.47284-9	1.47573-9
^{239}Pu	1.05749-5	1.05747-5	1.05744-5
^{240}Pu	9.95921-7	9.95925-7	9.95900-7
^{241}Pu	3.34194-7	3.34204-7	3.34495-7
^{242}Pu	1.63649-8	1.63651-8	1.64266-8
^{243}Pu	1.36278-11	1.36274-11	1.30074-11
^{241}Am	5.86427-10	5.86434-10	5.88396-10
^{242}Am	6.17245-12	6.17252-12	6.07897-12
^{242m}Am	5.04845-12	5.04856-12	5.07729-12
^{243}Am	4.56845-10	4.56857-10	4.61643-10
^{244}Am	6.37716-14	6.37420-14	5.89385-14
^{242}Cm	5.49980-11	5.49992-11	5.56397-11
^{243}Cm	1.15385-13	1.15390-13	1.17834-13
^{244}Cm	2.06829-11	2.06835-11	2.11112-11
^{245}Cm	2.43419-13	2.43427-13	2.51613-13
^{135}I	8.82753-9	8.82752-9	8.81797-9
^{135}Xe	9.14759-10	9.14758-10	9.13807-10
^{147}Nd	1.21159-7	1.21157-7	1.21155-7
^{147}Pm	2.01810-7	2.01815-7	2.01813-7
^{148}Pm	4.57034-9	4.57045-9	4.55895-9
^{148m}Pm	3.86715-9	3.86724-9	3.85696-9
^{149}Pm	1.99681-8	1.99682-8	1.99655-8
^{149}Sm	1.19776-8	1.19777-8	1.19778-8
Fission Products	1.45224-5	1.45228-5	1.45235-5
SUP	45.16	9.41	2.14 (seconds)

BENCHMARK PROBLEM SOLUTION

Identification: 15-A1-3 Benchmark Problem ID.15-A1

Date Submitted: June 1976 By: M. R. Wagner (KWU)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
R. R. Lee (CE)

Descriptive Title: Two-group Constant Parameter Point Depletion Problem

Mathematical Model

For each time-step the reaction rates, which are the entries of the nuclide depletion matrix $\underline{\underline{A}}$ are assumed constant. Furthermore, the matrix $\underline{\underline{A}}$ is assumed to be a lower triangular matrix which can be decomposed into

1. a number of lower triangular submatrices corresponding to independent heavy nuclide and fission product chains,

2. a rectangular matrix which provides the coupling of the fission product chains to the heavy nuclide chains.

Both analytic and third-order difference solutions are defined.

Pertinent Features of Solution Method

The code automatically investigates the structure of the depletion matrix and separates the independent fuel and fission product chains. The depletion equations for the fuel chains are solved first. A constant fission product source vector $\underline{S}$ is then computed

$$S_i = \sum_{j=1}^{N_{fuel}} Y_{ij} \; \overline{N}_j^{\Delta t} \sum_g \sigma_{fj}^g \; \Phi^g$$

where S_i is the component for fission product i, to replace the actual time-dependent rates $S_i(t)$ of fission product formation. Finally the decoupled fission product depletion chains are solved for that source.

In solving the depletion chains the code decides internally for every time-step and each nuclide whether the analytic solution or difference approximation will be applied. An input error criterion ε is used and by solving the equation

$$e^{-\alpha} - \frac{2 - \alpha}{2 + \alpha} = \varepsilon$$

a critical value α for the magnitude of the removal term $|A_{ii}|\ \Delta t$ is determined. If

$$|A_{ii}|\ \Delta t > \varepsilon$$

an analytic solution is requested for nuclide i.

Table 1 shows that the analytic solution ($\varepsilon \rightarrow 0$) deteriorates with reduced time-step lengths due to buildup of round-off errors for very long depletion chains.

Table 2 illustrates that the error buildup is reduced when using a combination of analytic and difference solutions.

Table 3 shows the dependence of the solution on the parameter ε for a time step length of $\Delta t = 2$ days

Column 1 of Table 2 is considered the best solution of this series.

Computer: CYBER 175 Date Solved: May 1976
at: KWU

Program: MEDIUM-2

Type of Program: Multidimensional LWR depletion code

References

1. M.R. Wagner, GAUGE, A Two-Dimensional Few-Group Neutron Diffusion-Depletion Program for a Uniform Triangular Mesh, GA 8307, Gulf General Atomic (1968)

2. M.R. Wagner, MEDIUM, ein mehrdimensionales Abbrandprogramm für Druckwasserreaktoren, Reaktortagung 1972 des Deutschen Atomforums/KTG, Hamburg, April 1972, Proceedings p. 161

Table 1. MEDIUM-2 Solutions to Constant-Parameter Problem
atoms/ (barn-cm) at 50 days
Analytic Solutions, $\varepsilon = 10^{-20}$

	Time Steps		
	10 min.	1 hour	2 days
U 235	.583394E-04	.583394E-04	.583394E-04
U 236	.286058E-05	.286058E-05	.286058E-05
U 237	.206103E-07	.206103E-07	.206103E-07
U 238	.691924E-02	.691924E-02	.691924E-02
U 239	.718370E-08	.718370E-08	.718370E-08
Np 237	.450845E-07	.450845E-07	.450845E-07
Np 238	.324894E-09	.324894E-09	.324894E-09
Np 239	.102945E-05	.102945E-05	.102945E-05
Np 240	.132294E-10	.132294E-10	.132294E-10
Pu 238	.147282E-08	.147283E-08	.147283E-08
Pu 239	.105747E-04	.105747E-04	.105747E-04
Pu 240	.995928E-06	.995928E-06	.995928E-06
Pu 241	.334210E-06	.334206E-06	.334205E-06
Pu 242	.163653E-07	.163651E-07	.163651E-07
Pu 243	.136281E-10	.136280E-10	.136280E-10
Am 241	.586454E-09	.586437E-09	.586433E-09
Am 242	.617274E-11	.617255E-11	.617251E-11
Am 242m	.504883E-11	.504860E-11	.504854E-11
Am 243	.456904E-09	.456855E-09	.456858E-09
Am 244	.637807E-13	.637740E-13	.637743E-13
Cm 242	.550017E-10	.549986E-10	.549984E-10
Cm 243	.113558E-12	.115114E-12	.115372E-12
Cm 244	.203962E-10	.206754E-10	.206821E-10
Cm 245	.199554E-12	.241771E-12	.243340E-12
I 135	.882757E-08	.882757E-08	.881793E-08
Xe 135	.914763E-09	.914763E-09	.913803E-09
Nd 147	.121157E-06	.121157E-06	.121154E-06
Pm 147	.201815E-06	.201815E-06	.201816E-06
Pm 148	.457044E-08	.457044E-08	.457048E-08
Pm 148m	.386724E-08	.386724E-08	.386727E-08
Pm 149	.199683E-07	.199683E-07	.199657E-07
Sm 149	.119777E-07	.119777E-07	.119776E-07
Fission Products	.145228E-04	.145228E-04	.145228E-04
CPU time, sec CYBER 175	88.94	15.74	0.39

Table 2. MEDIUM-2 Solutions to Constant Parameter Problem
atoms/ (barn-cm) at 50 days
Combination of Analytic and Difference Solutions $\varepsilon=10^{-5}$

	Time Steps		
	10 min.	1 hour	2 days
U 235	.583394E-04	.583394E-04	.583393E-04
U 236	.286058E-05	.286058E-05	.286059E-05
U 237	.206103E-07	.206103E-07	.205936E-07
U 238	.691924E-02	.691924E-02	.691924E-02
U 239	.718370E-08	.718371E-08	.718404E-08
Np 237	.450845E-07	.450845E-07	.450929E-07
Np 238	.324894E-09	.324894E-09	.325096E-09
Np 239	.102945E-05	.102945E-05	.102946E-05
Np 240	.132294E-10	.132294E-10	.132295E-10
Pu 238	.147283E-08	.147284E-08	.147572E-08
Pu 239	.105747E-04	.105747E-04	.105744E-04
Pu 240	.995928E-06	.995929E-06	.995900E-06
Pu 241	.334205E-06	.334206E-06	.334496E-06
Pu 242	.163651E-07	.163652E-07	.164267E-07
Pu 243	.136280E-10	.136275E-10	.130075E-10
Am 241	.586433E-09	.586435E-09	.588397E-09
Am 242	.617251E-11	.617253E-11	.607898E-11
Am 242m	.504854E-11	.504857E-11	.507732E-11
Am 243	.456858E-09	.456861E-09	.461644E-09
Am 244	.637734E-13	.637426E-13	.589387E-13
Cm 242	.549983E-10	.549993E-10	.556398E-10
Cm 243	.115387E-12	.115390E-12	.117634E-12
Cm 244	.206834E-10	.206836E-10	.211114E-10
Cm 245	.243424E-12	.243428E-12	.251615E-12
I 135	.882757E-08	.882757E-08	.881797E-08
Xe 135	.914763E-09	.914763E-09	.913807E-09
Nd 147	.121157E-06	.121157E-06	.121155E-06
Pm 147	.201815E-06	.201815E-06	.201813E-06
Pm 148	.457044E-08	.457044E-08	.455895E-08
Pm 148m	.386724E-08	.386724E-08	.385697E-08
Pm 149	.199683E-07	.199683E-07	.199655E-07
Sm 149	.119777E-07	.119777E-07	.119778E-07
Fission Products	.145228E-04	.145228E-04	.145234E-04
CPU time, sec CYBER 175	36.40	7.26	0.23

Table 3. MEDIUM-2 Solutions to Constant Parameter Problem
atoms/ (barn-cm) at 50 days
Combination of Analytic and Difference Solutions, $\Delta T=2$ days

	$\varepsilon = 10^{-5}$ $\alpha = 0.050$	$\varepsilon = 10^{-6}$ $\alpha = 0.023$	$\varepsilon = 10^{-7}$ $\alpha = 0.0106$
U 235	.583393E-04	.583393E-04	.583393E-04
U 236	.286059E-05	.286059E-05	.286059E-05
U 237	.205936E-07	.205936E-07	.205936E-07
U 238	.691924E-02	.691924E-02	.691924E-02
U 239	.718404E-08	.718404E-08	.718404E-08
Np 237	.450929E-07	.450929E-07	.450929E-07
Np 238	.325096E-09	.325096E-09	.325096E-09
Np 239	.102946E-05	.102946E-05	.102946E-05
Np 240	.132295E-10	.132295E-10	.132295E-10
Pu 238	.147572E-08	.147582E-08	.147582E-08
Pu 239	.105744E-04	.105747E-04	.105747E-04
Pu 240	.995900E-06	.995927E-06	.995927E-06
Pu 241	.334496E-06	.334205E-06	.334205E-06
Pu 242	.164267E-07	.163640E-07	.163651E-07
Pu 243	.130075E-10	.129556E-10	.136280E-10
Am 241	.588397E-09	.586297E-09	.586432E-09
Am 242	.607898E-11	.605585E-11	.617250E-11
Am 242m	.507732E-11	.504976E-11	.504853E-11
Am 243	.461644E-09	.458475E-09	.456857E-09
Am 244	.589387E-13	.585170E-13	.637742E-13
Cm 242	.556398E-10	.552501E-10	.549979E-10
Cm 243	.117834E-12	.116581E-12	.115757E-12
Cm 244	.211114E-10	.208856E-10	.206816E-10
Cm 245	.251615E-12	.247885E-12	.244501E-12
I 135	.881797E-08	.881792E-08	.881792E-08
Xe 135	.913807E-09	.913802E-09	.913802E-09
Nd 147	.121155E-06	.121154E-06	.121154E-06
Pm 147	.201813E-06	.201816E-06	.201816E-06
Pm 148	.455895E-08	.457048E-08	.457048E-08
Pm 148m	.385697E-08	.386727E-08	.386727E-08
Pm 149	.199655E-07	.199657E-07	.199657E-07
Sm 149	.119778E-07	.119776E-07	.119776E-07
Fission Products	.145234E-04	.145228E-04	.145228E-04
CPU time, sec CYBER 175	0.23	0.237	0.279

BENCHMARK PROBLEM SOLUTION

Identification: 15-A1-4

Date Submitted: January 1977

Date Accepted: June 1977

Benchmark Problem ID.15-A1

By: G. W. Cunningham and D. R. Vondy (ORNL)

By: H. L. Dodds, Jr. (U. of Tenn.) R. R. Lee (CE)

Descriptive Title: Two-Group Constant Parameter Point Depletion Problem

Computer: IBM-360/91

Code: BURNER module of the system containing the VENTURE neutronics code (Report ORNL-5180)

Date Solved: Dec. 1976 at ORNL

Results: Primary results are shown in Table 1 for benchmark quality calculations applying the matrix exponential method and the difference solution using average generation rates, to the linear chain equations. Other results have been obtained, as to test the explicit chain solution and criteria on series termination and dependence of the results on steps taken, which could be made available. These calculations were done with minimum computer memory use; only the direct coupling terms were stored, not the matrices, a procedure of calculation which is not directed toward minimizing processor time. No assumptions about equilibrium were used in obtaining the results. Calculations were done in double-precision (IBM long word).

TABLE 1. ORNL SOLUTIONS TO CONSTANT-PARAMETER PROBLEM, ATOMS/(BARN-CM) AT 50 DAYS

NUCLIDE	MATRIX EXPONENTIAL	AVERAGE GENERATION RATE		
	(570 STEPS)	(24 STEPS)	(768 STEPS)	(1536 STEPS)
U 235	5.83394-05	5.83394-05	5.83394-05	5.83394-05
U 236	2.86058-06	2.86061-06	2.86058-06	2.86058-06
U 237	2.06102-08	2.05921-08	2.06102-08	2.06102-08
U 238	6.91923-03	6.91923-03	6.91923-03	6.91923-03
U 239	7.18369-09	7.18404-09	7.18369-09	7.18369-09
NP237	4.50843-08	4.50166-08	4.50842-08	4.50843-08
NP238	3.24893-10	3.22611-10	3.24891-10	3.24892-10
NP239	1.02945-06	1.02951-06	1.02945-06	1.02945-06
NP240	1.32294-11	1.32307-11	1.32294-11	1.32294-11
PU238	1.47282-09	1.46092-09	1.47281-09	1.47282-09
PU239	1.05747-05	1.03861-05	1.05725-05	1.05741-05
PU240	9.95924-07	9.60185-07	9.95490-07	9.95805-07
PU241	3.34204-07	3.14777-07	3.33954-07	3.34134-07
PU242	1.63650-08	1.50433-08	1.63467-08	1.63599-08
PU243	1.36279-11	1.18517-11	1.36113-11	1.36232-11
AM241	5.86430-10	5.39966-10	5.85792-10	5.86249-10
AM242	6.17244-12	5.55259-12	6.16540-12	6.17045-12
AM242M	5.04867-12	4.57492-12	5.04189-12	5.04674-12
AM243	4.56854-10	3.83676-10	4.56117-10	4.56645-10
AM244	6.37737-14	4.85960-14	6.35965-14	6.37245-14
CM242	5.49975-11	4.81328-11	5.49124-11	5.49732-11
CM243	1.15385-13	9.88914-14	1.15162-13	1.15321-13
CM244	2.06831-11	1.49807-11	2.06116-11	2.06631-11
CM245	2.43421-13	1.69228-13	2.42425-13	2.43141-13
I 135	8.82755-09	8.76837-09	8.82697-09	8.82740-09
XE135	9.14763-10	9.06947-10	9.14688-10	9.14743-10
CS135	7.71695-08	7.39781-08	7.71478-08	7.71639-08
ND147	1.21157-07	1.20533-07	1.21150-07	1.21155-07
PM147	2.01814-07	2.00596-07	2.01801-07	2.01811-07
PM148	4.57044-09	4.52948-09	4.57012-09	4.57035-09
PM148M	3.86722-09	3.83199-09	3.86695-09	3.86715-09
PM149	1.99682-08	1.98139-08	1.99669-08	1.99679-08
SM149	1.19777-08	1.18613-08	1.19769-08	1.19775-08
FPLL	1.45228-05	1.44349-05	1.45217-05	1.45225-05
IBM 360/91 CPU TIME (SEC)	14.19	4.79	9.87	14.04

BENCHMARK PROBLEM

Identification: 15-A2 Source Situation ID.15
Date Submitted: July 1976 By: M. V. Gregory (SRL)
Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
R. R. Lee (CE)

Descriptive Title

Two-Group Constant Parameter Point Depletion Problem with Feedback

Reduction from Source Situation

1. Identical to Problem 15-A1 with the exception that α-decay and β^+-decay are NOT excluded, thus matrix $\underline{\underline{A}}$ is not simply triangular.

2. The additional data required for the problem with feedback are given in Table C.2a. Remaining data same as in Problem 15-A1.

Expected Primary Results

1. 50-day concentrations
2. Calculational statistics

Solutions Available

1. Fourth-order Runge-Kutta integration of depletion equations: 15-A2-1
2. Matrix exponential method and finite difference solution: 15-A2-2

The above two solutions are identical to four significant places with the smallest time-step.

BENCHMARK PROBLEM SOLUTION

Identification:	15-A2-1	Benchmark Problem ID 15-A2
Date Submitted:	July 1976	By: M. V. Gregory (SRL)
Date Accepted:	June 1977	By: H. L. Dodds, Jr. (U. of Tenn.)
		R. R. Lee (CE)

Descriptive Title

Two-Group Constant Parameter Point Depletion Problem with Feedback

Technique Used

Identical to solution of Problem ID 15-A1 without feedback, using atypically small integration time-step of 1 minute.

Computer

IBM-360/195

Program

BURGR

References

1. *The JOSHUA System, Volume 10.1 Generalized Reactor Analysis Subsystem: Physics.* (Draft) DPSTM-500 (1975)

2. M. M. Anderson et al., "Three-Dimensional Coupled Neutronics and Engineering Calculation of Savannah River Reactors," CONF-750413, Vol. II., p. VI-123.

Primary Results

The isotopic inventory at 50 days is given in Table C.2b. A fixed time-step of 1 minute was used for the Runge-Kutta integration. The CPU time required was 430 seconds. BURGR was constrained to such a small time-step size for the specific purpose of obtaining a benchmark-quality solution, thus the long running time should not be considered typical of production runs.

TABLE C.2a. Extra Data Required for Full-Chain Problem

Decay Constants

Isotope	Emitted Particle	Constant, sec^{-1}
^{238}Pu	α	2.550E-10
^{241}Am	α	5.090E-11
^{242}Am	β^+	2.030E-6
^{242}Cm	α	4.910E-8
^{243}Cm	α	6.860E-10
^{244}Cm	α	1.250E-9

(n,2n) Microscopic Cross Sections
(Group 1 only)

Isotope	σ, barns
^{235}U	0.002603
^{238}U	0.0043972
^{237}Np	0.00020144

TABLE C.2b. SRL Solution to Full-Chain Constant Parameter Point Depletion Problem, atoms/barn-cm at 50 days, Integration Time-Step of 1 minute

Isotope	Concentration	Isotope	Concentration
^{234}U	0.428817E-9	^{241}Am	0.586390E-9
^{235}U	0.583393E-4	^{242}Am	0.520984E-11
^{236}U	0.286054E-5	^{242m}Am	0.504819E-11
^{237}U	0.356768E-7	^{243}Am	0.457037E-9
^{238}U	0.691916E-2	^{244}Am	0.637996E-13
^{239}U	0.718357E-8	^{242}Cm	0.449667E-10
^{237}Np	0.104736E-6	^{243}Cm	0.950949E-13
^{238}Np	0.780485E-9	^{244}Cm	0.206737E-10
^{239}Np	0.102944E-5	^{245}Cm	0.243318E-12
^{240}Np	0.132292E-10	^{135}I	0.882738E-8
^{238}Pu	0.441854E-8	^{135}Xe	0.914753E-9
^{239}Pu	0.105748E-4	^{147}Nd	0.121154E-6
^{240}Pu	0.995892E-6	^{147}Pm	0.201810E-6
^{241}Pu	0.334195E-6	^{148}Pm	0.457029E-8
^{242}Pu	0.163736E-7	^{148m}Pm	0.386727E-8
^{243}Pu	0.136348E-10	^{149}Pm	0.199679E-7
		^{149}Sm	0.119776E-7
		Fission Products	0.145225E-4

BENCHMARK PROBLEM SOLUTION

Identification: 15-A2-2

Benchmark Problem ID.15-A2

Date Submitted: January 1977

By: G. W. Cunningham and D. R. Vondy (ORNL)

Date Accepted: June 1977

By: H. L. Dodds, Jr. (U. of Tenn.)
R. R. Lee (CE)

Descriptive Title: Solutions for the Constant Parameter Depletion Problem with Feedback

Computer: IBM-360/91

Code: BURNER module of the system containing the VENTURE neutronics code (Report ORNL-5180)

Date Solved: December 1976 at ORNL

Results: Primary results are shown in Table 1 for benchmark quality calculations applying the matrix exponential method and the difference solution using average generation rates, to the linear chain equations with feedback. Other results have been obtained, as to test the explicit chain solution and criteria on series termination and dependence of the results on steps taken, which could be made available. These calculations were done with minimum computer memory use; only the direct coupling terms were stored, not the matrices, a procedure of calculation which is not directed toward minimizing processor time. Calculations were done in double-precision (IBM long word).

Table 1. Nuclide Concentrations for the Constant Parameter Point Depletion Problem with Feedback

Nuclide	Matrix Exponential (570 steps)	Average Generation Rate (24 steps)	(768 steps)	(1536 steps)
^{234}U	.428821-9	.43280-9	.42895-9	.42888-9
^{235}U	.583390-4	.58339-4	.58339-4	.58339-4
^{236}U	.286057-5	.28606-5	.28606-5	.28606-5
^{237}U	.356780-7	.35661-7	.35678-7	.35678-7
^{238}U	.691915-2	.69191-2	.69191-2	.69191-2
^{239}U	.718360-8	.71840-8	.71836-8	.71836-8
^{237}Np	.104739-6	.10462-6	.10474-6	.10474-6
^{238}Np	.780515-9	.77641-9	.78051-9	.78052-9
^{239}Np	.102944-5	.10295-5	.10294-5	.10294-5
^{240}Np	.132292-10	.13231-10	.13229-10	.13229-10
^{238}Pu	.441870-8	.43857-8	.44187-8	.44187-8
^{239}Pu	.105747-4	.10386-4	.10573-4	.10574-4
^{240}Pu	.995926-6	.96019-6	.99549-6	.99580-6
^{241}Pu	.334204-6	.31478-6	.33395-6	.33413-6
^{242}Pu	.163743-7	.15050-7	.16356-7	.16369-7
^{243}Pu	.136356-10	.11857-10	.13619-10	.13631-10
^{241}Am	.586404-9	.53994-9	.58577-9	.58622-9
^{242}Am	.520999-11	.46704-11	.52040-11	.52083-11
^{242m}Am	.504831-11	.45746-11	.50415-11	.50464-11
^{243}Am	.457064-9	.38380-9	.45632-9	.45685-9
^{244}Am	.638030-13	.48611-13	.63625-13	.63754-13
^{242}Cm	.449681-10	.39205-10	.44899-10	.44948-10
^{243}Cm	.950979-13	.81087-13	.94915-13	.95045-13
^{244}Cm	.206748-10	.14973-10	.20603-10	.20655-10
^{245}Cm	.243334-12	.16915-12	.24234-12	.24305-12
^{135}I	.882752-8	.87683-8	.88269-8	.88274-8
^{135}Xe	.914759-9	.90694-9	.91468-9	.91474-9
^{135}Cs	.771693-7	.73978-7	.77148-7	.77164-7
^{147}Nd	.121156-6	.12053-6	.12115-6	.12115-6
^{147}Pm	.201814-6	.20060-6	.20180-6	.20181-6

Table 1 (continued)

Nuclide	Matrix Exponential (570 steps)	Average Generation Rate (24 steps)	(768 steps)	(1536 steps)
^{148}Pm	.457042-8	.45295-8	.45701-8	.45703-8
^{148m}Pm	.386722-8	.38320-8	.38669-8	.38671-8
^{149}Pm	.199682-7	.19814-8	.19967-7	.19968-7
^{149}Sm	.119776-7	.11861-7	.11977-7	.11977-7
F.P.	.145227-4	.14435-4	.14522-4	.14522-4
Processor Time (min)	0.18	0.02	0.11	0.19