ANL-7416 Supplement 2
Mathematics and Computers
(UC-32)

ARGONNE CODE CENTER: BENCHMARK PROBLEM BOOK

Prepared by the
Computational Benchmark Problems Committee of the
MATHEMATICS AND COMPUTATION DIVISION
OF THE AMERICAN NUCLEAR SOCIETY

Revised June 1977

Benchmark Problems Included

11. Multi-dimensional (x-y-z) LWR Model
13. Neutron Transport in a BWR Rod Bundle
14. Multi-dimensional (x-y-z) BWR Model
15. Neutronic Depletion Benchmark Problems

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ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

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BENCHMARK SOURCE SITUATION

Identification: 14

Date Submitted: June 1976 By: S. Langenbuch (GRS-Munich)
W. Werner (GRS-Munich)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
F. N. McDonnell (AECL-CRNL)

Descriptive Title: Multi-dimensional (x-y-z) BWR Model

Suggested Function: Test 2d, 3d Neutron Kinetics Solution, Especially for Coarse Mesh Methods

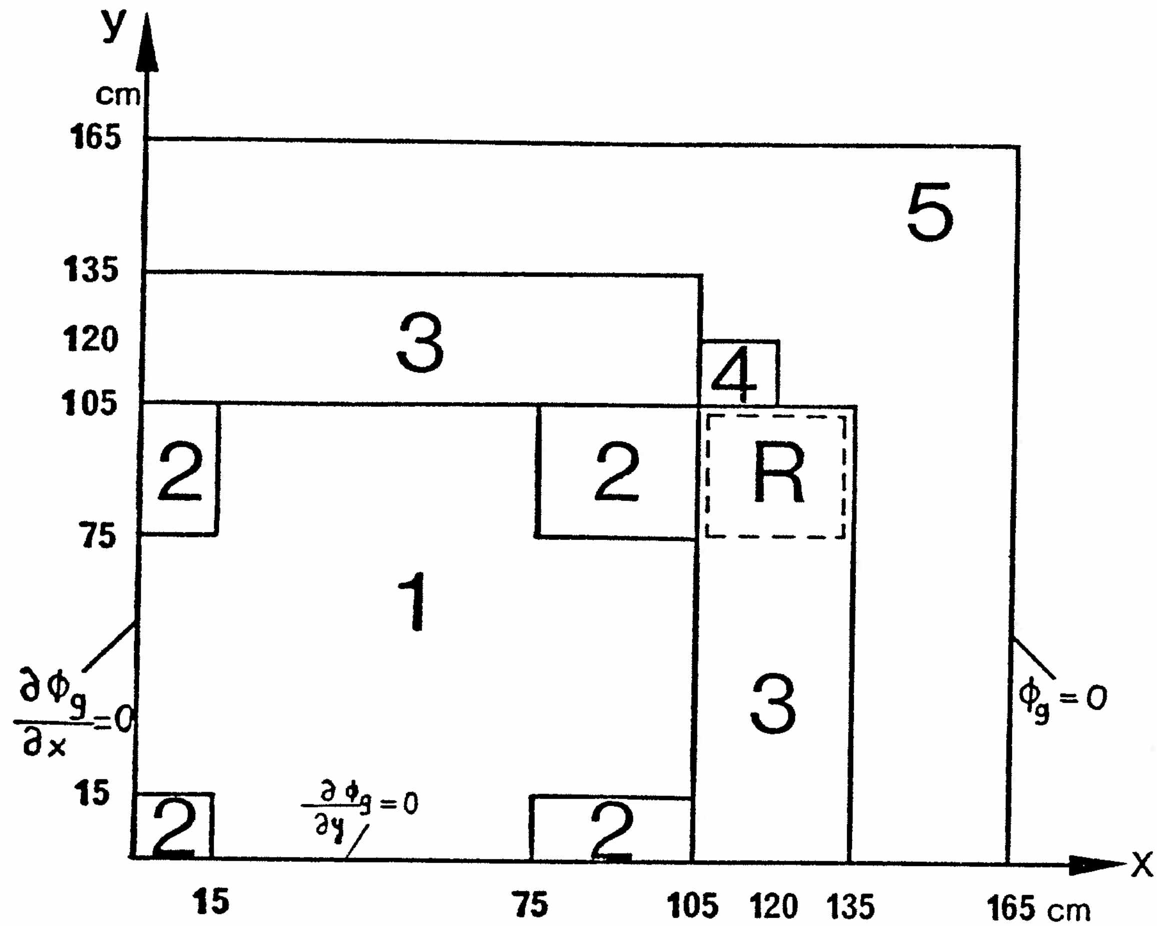

Fig. 1: Quadrant of Reactor Horizontal Cross Section, Region Assignment

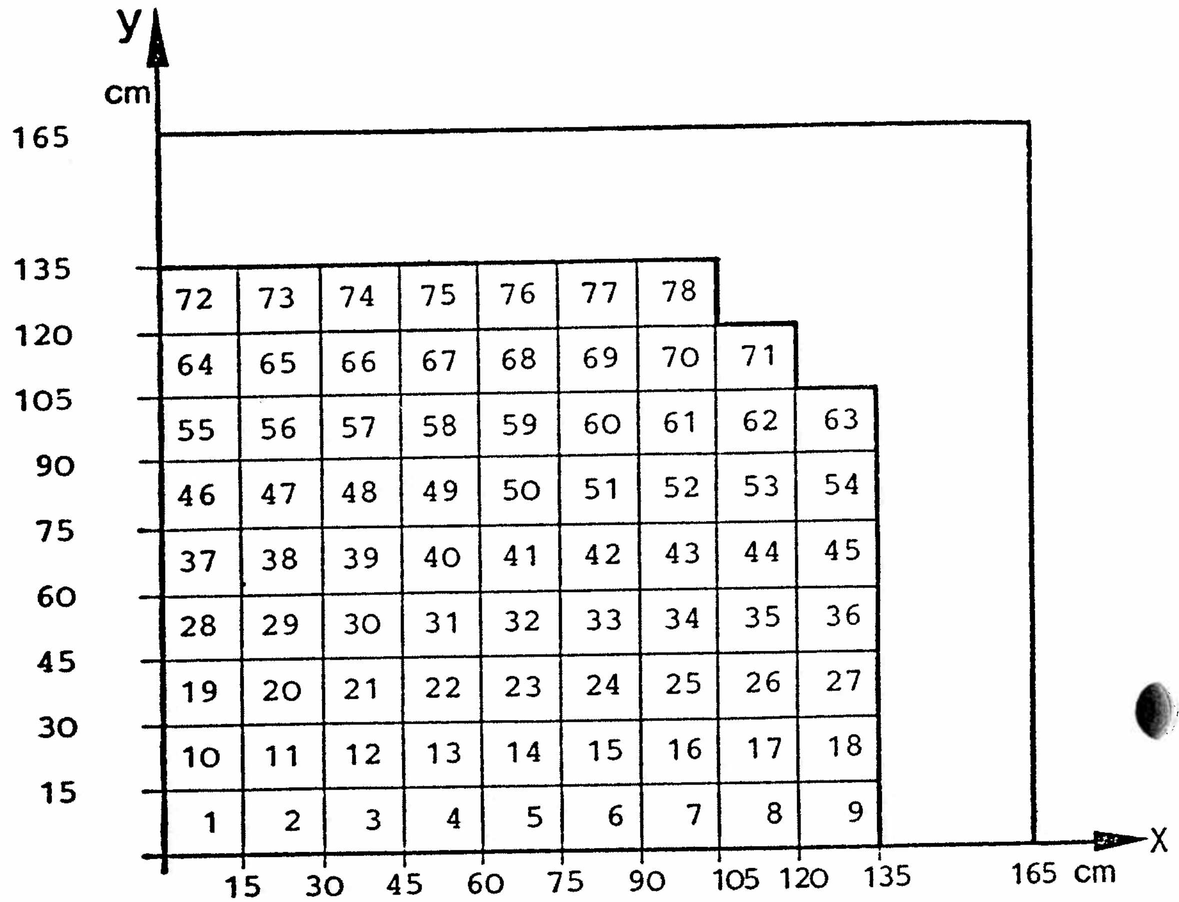

Fig. 2: Quadrant of Reactor Horizontal Cross Section, Fuel Assembly Identification

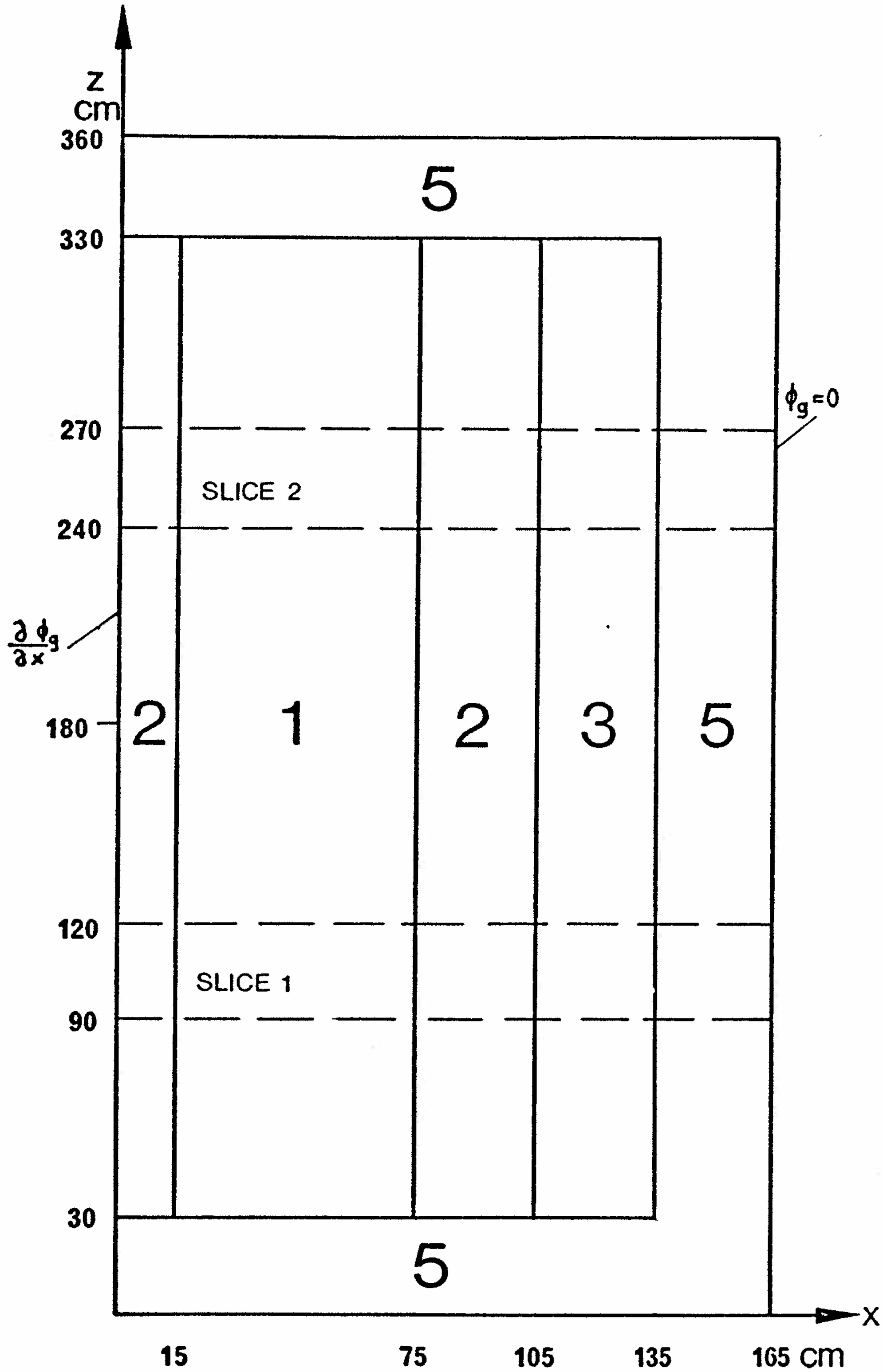

Fig. 3: Vertical Cross Section, y=0, Region Assignment, Vertical Slice Identification

BOUNDARY CONDITIONS:
External Boundaries: zero flux
Symmetry Boundaries: Reflection, $\frac{\partial \phi_g}{\partial n} = 0$

BENCHMARK PROBLEM

Identification: 14-A1 Source Situation ID.14

Date Submitted: June 1976 By: S. Langenbuch and W. Werner (GRS-Munich)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.) F. N. McDonnell (AECL-CRNL)

Descriptive Title: Super Prompt-critical Transient; Two-dimensional, Two-group Neutron Diffusion Problem, with Adiabatic Heatup and Doppler Feedback in Thermal Reactor

Reduction of Source Situation:

1. Two-dimensional (xy), two-group diffusion theory
2. Two delayed neutron precursor groups.

$$\nabla D_1(\vec{x},t)\nabla\phi_1(\vec{x},t)-(\Sigma a_1(\vec{x},t)+\Sigma_{1\to2}(\vec{x},t))\phi_1(\vec{x},t)+\nu(1-\beta)\left[\Sigma f_1(\vec{x},t)\phi_1(\vec{x},t)\right.$$

$$\left.+\Sigma f_2(\vec{x},t)\phi_2(\vec{x},t)\right]+\sum_{i=1}^{2}\lambda_i C_i(\vec{x},t) = \frac{1}{v_1}\frac{\partial}{\partial t}\phi_1(\vec{x},t)$$

$$\nabla D_2(\vec{x},t)\nabla\phi_2(\vec{x},t)-\Sigma a_2(\vec{x},t)\phi_2(\vec{x},t)+\Sigma_{1\to2}(\vec{x},t)\phi_1(\vec{x},t) = \frac{1}{v_2}\frac{\partial}{\partial t}\phi_2(\vec{x},t)$$

$$\nu\beta_i(\Sigma f_1(\vec{x},t)\phi_1(\vec{x},t)+\Sigma f_2(\vec{x},t)\phi_2(\vec{x},t))-\lambda_i C_i(\vec{x},t) = \frac{\partial}{\partial t}C_i(\vec{x},t),\ i=1,2.$$

with zero flux boundary conditions on external surfaces, reflection conditions at symmetry boundaries, and steady state initial conditions.

3. Adiabatic Heatup

$$\alpha\left[\Sigma f_1(\vec{x},t)\phi_1(\vec{x},t)+\Sigma f_2(\vec{x},t)\phi_2(\vec{x},t)\right] = \frac{\partial}{\partial t}T(\vec{x},t)$$

4. Doppler Feedback

$$\Sigma a_1(\vec{x},t) = \Sigma a_1(\vec{x},t=0)\left[1+\gamma\left(\sqrt{T(x,t)}-\sqrt{T_o}\right)\right]$$

5. Power

$$P(\vec{x},t) = \varepsilon\left[\Sigma f_1(\vec{x},t)\phi_1(\vec{x},t)+\Sigma f_2(\vec{x},t)\phi_2(\vec{x},t)\right]$$

Data:

Initial Two-Group Constants

Region	Material	Group i	D_i(cm)	Σa_i(cm^{-1})	$\nu\Sigma f_i$(cm^{-1})	$\Sigma_{1\to2}$(cm^{-1})
1	Fuel 1 with rod	1 2	1.255 0.211	0.008252 0.1003	0.004602 0.1091	0.02533
2	Fuel 1 without rod	1 2	1.268 0.1902	0.007181 0.07047	0.004609 0.08675	0.02767
3	Fuel 2 with rod	1 2	1.259 0.2091	0.008002 0.08344	0.004663 0.1021	0.02617
4	Fuel 2 without rod	1 2	1.259 0.2091	0.008002 0.073324	0.004663 0.1021	0.02617
5	Reflector	1 2	1.257 0.1592	0.0006034 0.01911	0 0	0.04754

Additional Parameters for all Regions:

$B^2 = 1.0\cdot10^{-4}$ axial buckling for both energy groups

$\nu = 2.43$ mean number of neutrons per fission

$v_1 = 3.0\cdot10^{7}$ $cm\cdot sec^{-1}$

$v_2 = 3.0\cdot10^{5}$ $cm\cdot sec^{-1}$

Delayed Neutron Data:

Group	β_i	λ_i(sec^{-1})
1	0.0054	0.00654
2	0.001087	1.35

Data for Feedback Model

α	= 3.83	10^{-11} $^oK\ cm^3$	conversion factor
γ	= 2.034	10^{-3} $^oK^{1/2}$	feedback constant
ε	= 3.204	10^{-11} Wsec/p.fission	energy conversion factor

The initial configuration is made critical by dividing the production cross sections by k_{eff}. The initial flux distribution shall be normalized such that the average power density

$$\overline{P} = \frac{\varepsilon}{V_{core}} \int_{V_{core}} (\Sigma f_1 \phi_1 + \Sigma f_2 \phi_2)dV = 1.0 \cdot 10^{-6}\ W\ cm^{-3}$$

The initial precursor concentrations are in equilibrium with the initial critical flux distribution.

The initial temperature $T_o = 300\ ^oK$.

Initiating Perturbation:

$$\frac{\Sigma a_2(t)}{\Sigma a_2(0)} = \begin{cases} 1 - 0.0606184 \cdot t & t \leq 2 \\ 0.8787631 & t > 2 \end{cases}$$

where t = time (sec).

Expected Primary Results:

1. Maximum eigenvalue for initial flux distribution

2. Normalized Power Densities P_k for initial flux distribution:

$$P_k = \frac{\varepsilon}{V_k \overline{P}} \int_{V_k} (\Sigma f_1 \phi_1 + \Sigma f_2 \phi_2) dV, \quad V_k = \text{Volume of Fuel Assembly } k$$

$$k = 1,\ldots,78$$

3. Maximum eigenvalue for configuration of withdrawn rod for cold reactor (feedback effects neglected)

4. Average Power density $\overline{P}$ versus time

5. Normalized Power Densities P_k at t=0,4 sec, 0,8 sec, 1,2 sec, 1,4 sec, 2,0 sec, 3,0 sec.

6. Maximum of $\overline{P}$, time of occurence

7. Average temperature $\overline{T} = \frac{1}{V_{core}} \int_{V_{core}} T(\vec{x},t)\, dV$ versus time

8. Number of unknowns in the problem, number of time-steps, computing time, and computer used.

Possible additional results:

9. Table of temperatures in volumes V_k, k=1,...,78.

10. Dependence of results on spatial and temporal discretization.

Solutions:

Coarse-mesh finite-difference methods

1.	Flux expansion:	14-A1-1
2.	Nodal expansion:	14-A1-2

BENCHMARK PROBLEM SOLUTION

Identification: 14-A1-1 Benchmark Problem ID.14-A1

Date Submitted: February 1977 By: S. Langenbuch (GRS-Munich)
W. Werner (GRS-Munich)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
F. N. McDonnell (AECL-CRNL)

Descriptive Title: Direct 2d-Coarse Mesh Solution with CUBBOX.[1,2,3]

Mathematical Model: A 5-point difference operator with coupling coefficients derived from expansion of neutron flux into local polynomials is used for the approximation of the spatial differential operator,[1,2,3]. Time integration is performed by a combined ADE-ADI technique[4] with spectral matching[5], and frequency prediction from space-averaged kinetic equations[6].

Computer: IBM-360, Model 91

Code: CUBBOX[7] with 6-order nonseparated polynomials

Date Solved: February 1977 at Laboratorium für Reaktorregelung und Anlagensicherung (LRA)

References:

1 A. Birkhofer and W. Werner, Efficiency of Various Methods for the Analysis of Space-Time Kinetics, Proc. Conf. Mathematical Models and Computational Techniques for Analysis of Nuclear Systems, CONF-730414, Vol. 2, p. IX-31-41 (1973)

2 A. Birkhofer, S. Langenbuch and W. Werner, Coarse-Mesh Method for Space-Time Kinetics, Trans. Am. Nuc. Soc., 18, 153 (1974)

3 S. Langenbuch, W. Maurer and W. Werner, High Order Schemes for Neutron Kinetics Calculations, Based on Local Polynomial Approximation, to be publ. Nucl. Sci. Eng. Oct. 1977

4 S. Langenbuch and W. Werner, Implicit Matrix Decomposition Scheme for Coarse-Mesh Methods, Trans. Am. Nuc. Soc. 21, 224 (1974)

5 J. Devooght and E. Mund, A - Stable Algorithms for Neutron Kinetics, MRR 145, Proc. of the Joint NEACRP/CSNI Specialists' Meeting on New Developments in Three-Dimensional Neutron Kinetics, 21-71 (1975)

6 S. Langenbuch and W. Werner, Eine Methode zur Verbesserung der Zeitintegration in 3d Neutronenkinetik-Rechnungen durch eine Form der Periodenfaktorisierung, Proc. Reaktortangung (1976)

7 Program Description of the QUABOX/CUBBOX Code, Internal Report, LRA, D-8046 Garching

Results:

Uniform mesh width $\Delta x = \Delta y$ = 15 cm (11 x 11 intervals)

1. Maximum eigenvalue for initial flux distribution:
   $k_{eff} = 0.99633$
2. Exhibit A:
   Normalized Power Densities P_{ki} for initial flux distribution:

   $$P_{ki} = \frac{\varepsilon}{\bar{P} V_{ki}} \int_{V_{ki}} (\Sigma f_1 \phi_1 + \Sigma f_2 \phi_2) dV, \quad V_k = \text{Volume of Fuel}$$

   Assembly k

   $k = 1, \ldots, 78$
3. Maximum eigenvalue for configuration of withdrawn rod for cold reactor (feedback effects neglected):
   $k_{eff} = 1.01546$
4. Exhibit B:
   Average Power density $\bar{P}$ versus time
5. Exhibit C:
   Normalized Power Densities P_k at t = 0.4 sec, 0.8 sec, 1.2 sec, 1.4 sec, 2.0 sec, 3.0 sec.
6. Maximum of $\bar{P} = 5.734 \cdot 10^3$; time of occurrence = 1.421 sec.
7. Exhibit D:
   Average temperature $\bar{T} = \frac{1}{V_{core}} \int_{V_{core}} T(\vec{x},t) dV$ versus time
8. Number of unknowns in the problem 121 x (2 prompt neutron groups + 2 delayed precursor groups + temperature) = 605
   Number of time-steps: 1200
   Computing time: 180 sec on IBM-360/91
9. Exhibit E:
   Average temperatures in volumes V_k, k = 1,...,78, at t = 0.4 sec, 0.8 sec, 1.2 sec, 1.4 sec, 2.0 sec, 3.0 sec.

EXHIBIT A

NORMALIZED POWER DENSITY — INITIAL STATE

Y/X	1	2	3	4	5	6	7	8	9
1	0.6214	0.4438	0.4134	0.5107	0.7939	1.3984	1.6725	1.4769	0.9059
2	0.4438	0.4010	0.4092	0.4931	0.6708	0.9416	1.1512	1.2703	0.8538
3	0.4134	0.4092	0.4275	0.4958	0.6209	0.7794	0.9608	1.1655	0.8145
4	0.5107	0.4931	0.4958	0.5565	0.6816	0.8400	1.0173	1.2168	0.8427
5	0.7939	0.6708	0.6209	0.6816	0.8655	1.1570	1.3450	1.4185	0.9260
6	1.3984	0.9416	0.7794	0.8400	1.1570	1.8739	2.0743	1.6882	0.9580
7	1.6725	1.1512	0.9608	1.0173	1.3450	2.0743	2.1888	1.6176	0.8403
8	1.4769	1.2703	1.1655	1.2168	1.4185	1.6882	1.6176	1.3323	0.0
9	0.9059	0.8538	0.8145	0.8427	0.9260	0.9580	0.8403	0.0	0.0

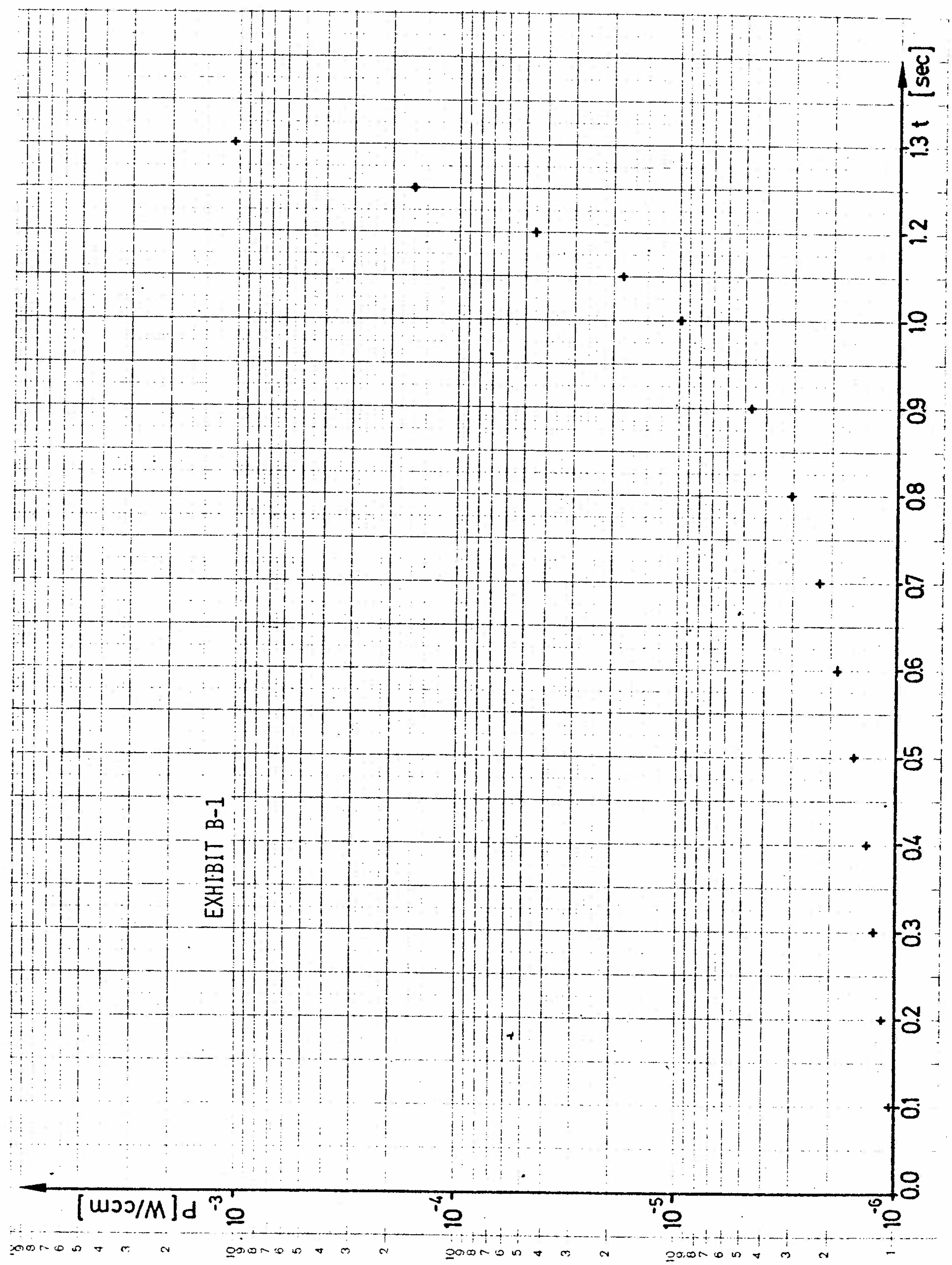

EXHIBIT B-1

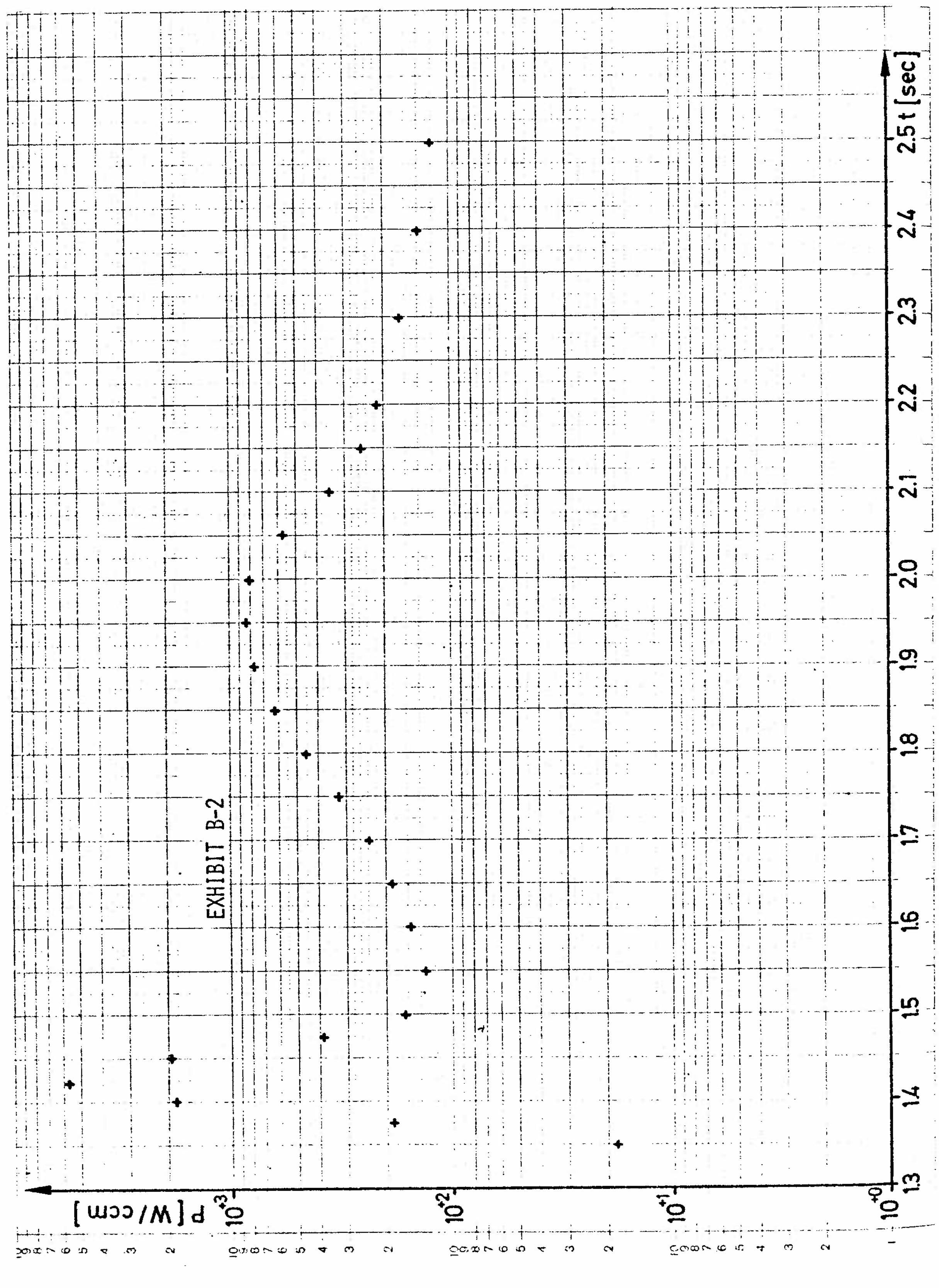
EXHIBIT B-2
t [sec]
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
P [W/ccm]
10^{+3}
10^{+2}
10^{+1}
10^{+0}

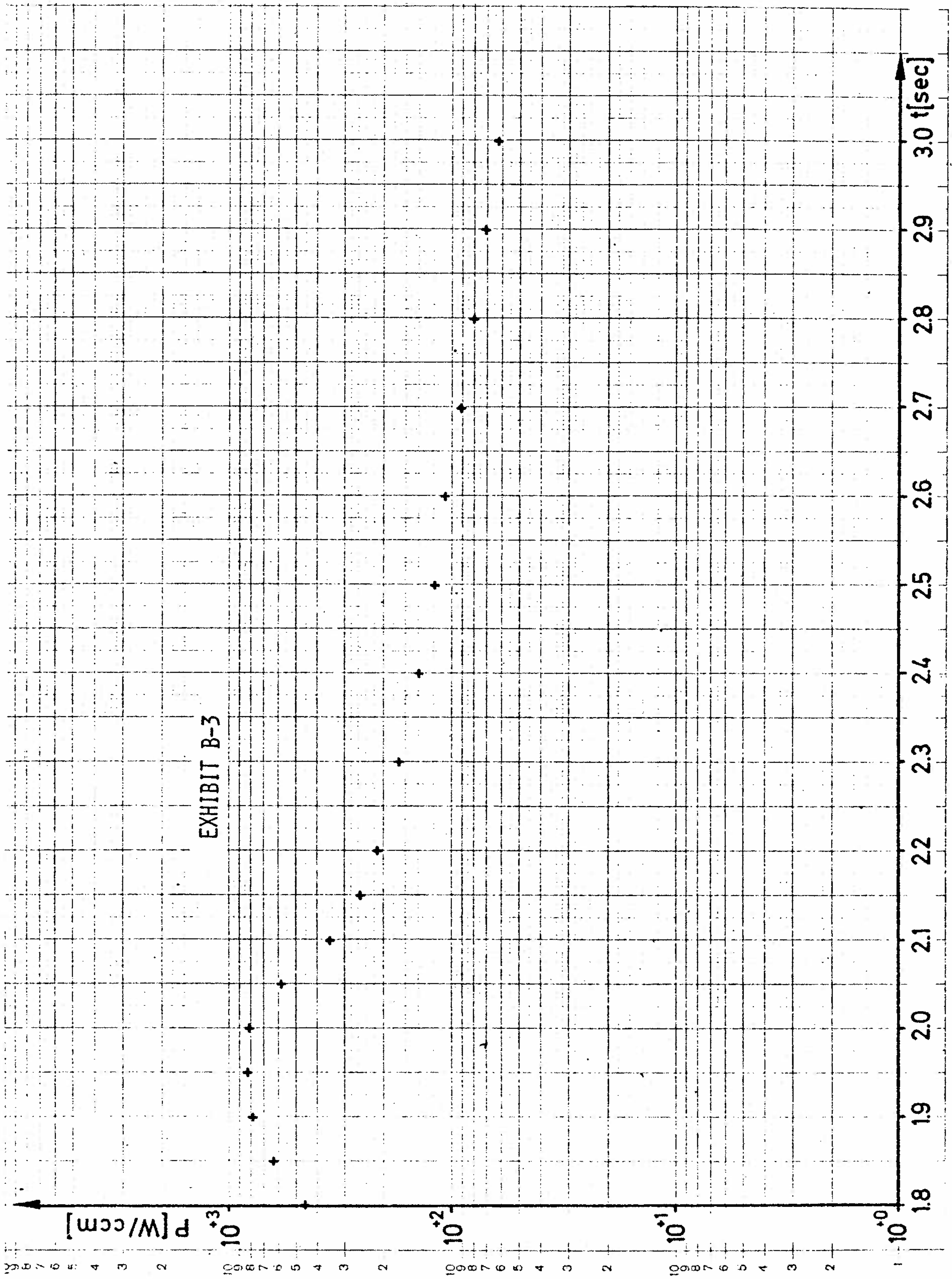
EXHIBIT B-3
t[sec]
P [W/ccm]
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3.0
10^{+3}
10^{+2}
10^{+1}
10^{+0}

EXHIBIT C-1
NORMALIZED POWER DENSITY

T = 0.4 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	0.5647	0.4056	0.3820	0.4770	0.7466	1.3193	1.5820	1.4007	0.8607
2	0.4046	0.3679	0.3799	0.4635	0.6358	0.8967	1.1012	1.2202	0.8221
3	0.3789	0.3778	0.4002	0.4718	0.5992	0.7603	0.9452	1.1526	0.8079
4	0.4697	0.4574	0.4679	0.5375	0.6733	0.8456	1.0398	1.2563	0.8747
5	0.7308	0.6230	0.5889	0.6659	0.8712	1.1956	1.4265	1.5428	1.0200
6	1.2869	0.8742	0.7405	0.8248	1.1757	1.9589	2.2457	1.9520	1.1405
7	1.5396	1.0692	0.9131	0.9985	1.3663	2.1688	2.3712	1.8785	1.0139
8	1.3604	1.1812	1.1071	1.1914	1.4330	1.7540	1.7310	1.4795	0.0
9	0.8349	0.7943	0.7734	0.8233	0.9313	0.9880	0.8876	0.0	0.0

EXHIBIT C-2

NORMALIZED POWER DENSITY T = 0.8 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	0.4776	0.3469	0.3342	0.4264	0.6764	1.2029	1.4492	1.2889	0.7945
2	0.3443	0.3172	0.3354	0.4190	0.5840	0.8309	1.0281	1.1474	0.7761
3	0.3255	0.3295	0.3587	0.4360	0.5676	0.7332	0.9238	1.1360	0.7999
4	0.4063	0.4023	0.4256	0.5094	0.6622	0.8559	1.0760	1.3182	0.9244
5	0.6331	0.5493	0.5401	0.6429	0.8815	1.2559	1.5507	1.7298	1.1617
6	1.1145	0.7702	0.6812	0.8026	1.2059	2.0899	2.5030	2.3413	1.4134
7	1.3344	0.9429	0.8402	0.9709	1.4004	2.3137	2.6446	2.2635	1.2739
8	1.1812	1.0443	1.0181	1.1538	1.4568	1.8550	1.9018	1.6987	0.0
9	0.7259	0.7033	0.7110	0.7945	0.9404	1.0345	0.9593	0.0	0.0

EXHIBIT C-3

NORMALIZED POWER DENSITY T = 1.2 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	0.3671	0.2724	0.2731	0.3610	0.5846	1.0491	1.2731	1.1409	0.7064
2	0.2677	0.2527	0.2783	0.3615	0.5162	0.7442	0.9315	1.0505	0.7144
3	0.2576	0.2677	0.3052	0.3893	0.5258	0.6970	0.8948	1.1127	0.7878
4	0.3248	0.3313	0.3705	0.4720	0.6462	0.8679	1.1225	1.3991	0.9891
5	0.5067	0.4537	0.4759	0.6110	0.8917	1.3312	1.7130	1.9807	1.3535
6	0.8903	0.6346	0.6025	0.7703	1.2395	2.2524	2.8393	2.8764	1.7966
7	1.0668	0.7777	0.7429	0.9302	1.4370	2.4917	2.9998	2.7914	1.6400
8	0.9472	0.8645	0.8985	1.0983	1.4784	1.9767	2.1196	1.9877	0.0
9	0.5832	0.5833	0.6266	0.7518	0.9456	1.0882	1.0475	0.0	0.0

EXHIBIT C-4

NORMALIZED POWER DENSITY

T = 1.4 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	0.3266	0.2449	0.2499	0.3348	0.5458	0.9816	1.1944	1.0743	0.6669
2	0.2395	0.2287	0.2565	0.3384	0.4876	0.7061	0.8879	1.0060	0.6861
3	0.2321	0.2443	0.2845	0.3703	0.5076	0.6801	0.8801	1.0996	0.7807
4	0.2935	0.3037	0.3484	0.4558	0.6374	0.8699	1.1396	1.4318	1.0157
5	0.4569	0.4157	0.4493	0.5956	0.8912	1.3562	1.7790	2.0942	1.4444
6	0.8004	0.5799	0.5690	0.7527	1.2442	2.3051	2.9778	3.1392	1.9948
7	0.9589	0.7103	0.7005	0.9070	1.4394	2.5457	3.1418	3.0478	1.8288
8	0.8527	0.7905	0.8458	1.0669	1.4731	2.0095	2.2000	2.1123	0.0
9	0.5256	0.5337	0.5892	0.7281	0.9383	1.1000	1.0766	0.0	0.0

EXHIBIT C-5

NORMALIZED POWER DENSITY T = 2.0 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	0.3031	0.2261	0.2287	0.3031	0.4893	0.8760	1.0653	0.9619	0.6025
2	0.2206	0.2101	0.2349	0.3086	0.4427	0.6399	0.8078	0.9237	0.6370
3	0.2105	0.2224	C.2604	0.3412	0.4710	0.6374	0.8363	1.0599	0.7632
4	0.2611	0.2726	0.3175	0.4226	0.6023	0.8412	1.1341	1.4628	1.0617
5	0.3991	0.3675	0.4059	0.5518	0.8483	1.3362	1.8421	2.2956	1.6435
6	0.6919	0.5064	0.5093	0.6933	1.1844	2.2846	3.1584	3.7559	2.5321
7	0.8242	0.6164	0.6226	0.8293	1.3607	2.5086	3.3176	3.6522	2.3560
8	0.7319	0.6862	0.7506	0.9721	1.3823	1.9583	2.2663	2.3526	0.0
9	0.4532	0.4655	0.5248	0.6649	0.8806	1.0662	1.0910	0.0	0.0

I

EXHIBIT C-6
NORMALIZED POWER DENSITY

T = 3.0 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	0.3333	0.2466	0.2454	0.3203	0.5115	0.9105	1.1035	0.9943	0.6228
2	0.2412	0.2277	0.2506	0.3241	0.4599	0.6604	0.8298	0.9461	0.6523
3	0.2279	0.2385	0.2749	0.3542	0.4827	0.6473	0.8443	1.0673	0.7685
4	0.2800	0.2896	0.3315	0.4330	0.6076	0.8399	1.1251	1.4476	1.0510
5	0.4254	0.3881	0.4206	0.5595	0.8459	1.3179	1.8054	2.2439	1.6073
6	0.7357	0.5335	0.5255	0.6992	1.1735	2.2410	3.0795	3.6504	2.4622
7	0.8748	0.6401	0.6413	0.8352	1.3466	2.4579	3.2311	3.5456	2.2883
8	0.7761	0.7206	0.7732	0.9805	1.3713	1.9225	2.2113	2.2894	0.0
9	0.4809	0.4890	0.5413	0.6723	0.8766	1.0508	1.0694	0.0	0.0

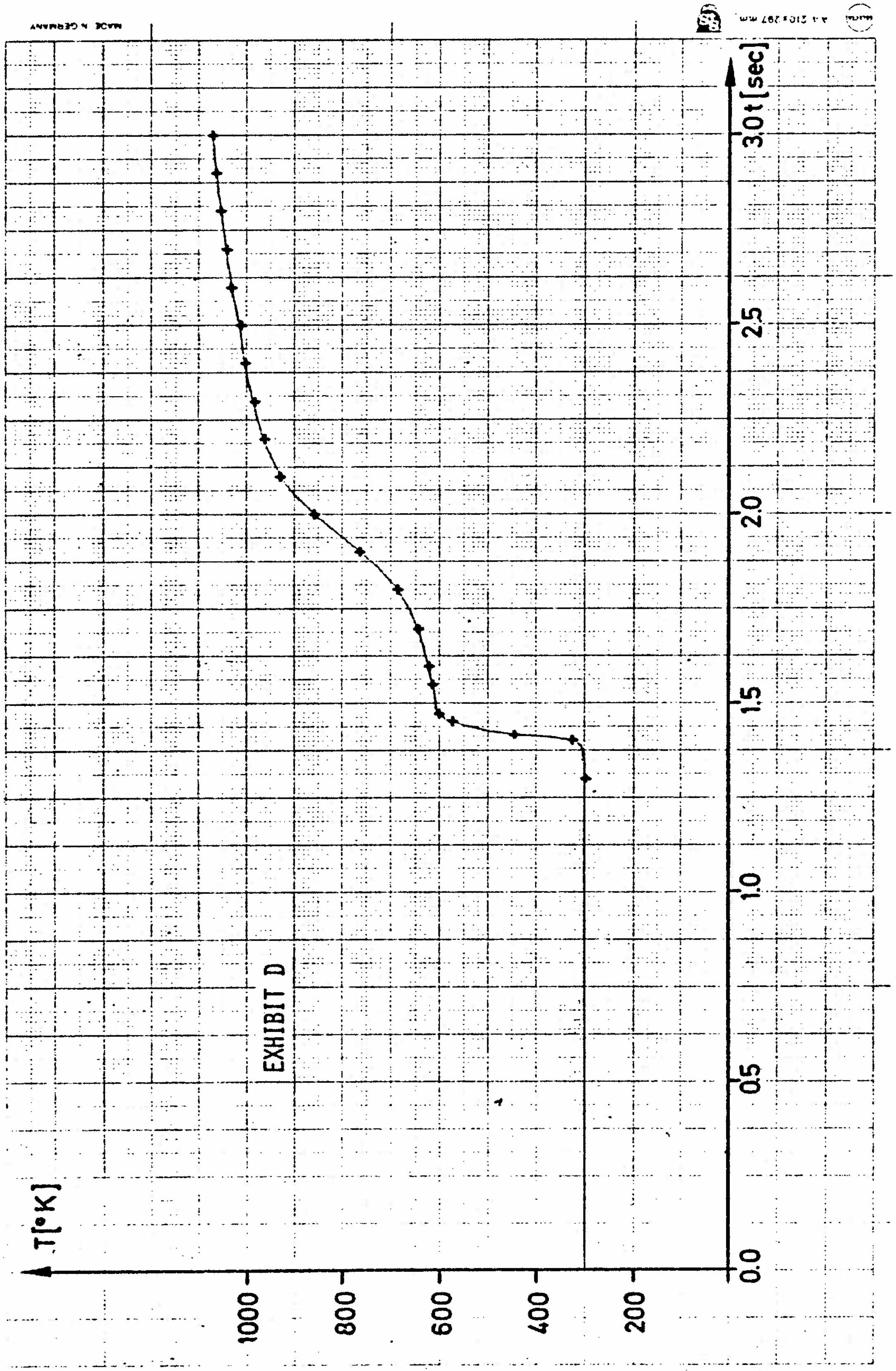
T[°K]
t[sec]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1000
800
600
400
200
EXHIBIT D
MADE IN GERMANY

EXHIBIT E-1

AVERAGE FUEL TEMPERATURE T = 0.4 SEC

VALUES ARE IDENTICAL TO 300.0

EXHIBIT E-2

AVERAGE FUEL TEMPERATURE T = 0.8 SEC

VALUES ARE IDENTICAL TO 300.0

EXHIBIT E-3
AVERAGE FUEL TEMPERATURE T = 1.2 SEC

VALUES ARE IDENTICAL TO 300.0

EXHIBIT E-4

AVERAGE FUEL TEMPERATURE T = 1.4 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	3.0838D+02	3.0630D+02	3.0642D+02	3.0861D+02	3.1406D+02	3.2524D+02	3.3071D+02	3.2766D+02	3.1716D+02
2	3.0616D+02	3.0587D+02	3.0659D+02	3.0870D+02	3.1252D+02	3.1818D+02	3.2287D+02	3.2585D+02	3.1764D+02
3	3.0596D+02	3.0628D+02	3.0731D+02	3.0951D+02	3.1305D+02	3.1749D+02	3.2263D+02	3.2827D+02	3.2006D+02
4	3.0754D+02	3.0781D+02	3.0895D+02	3.1171D+02	3.1639D+02	3.2236D+02	3.2929D+02	3.3680D+02	3.2611D+02
5	3.1176D+02	3.1067D+02	3.1155D+02	3.1531D+02	3.2287D+02	3.3491D+02	3.4579D+02	3.5374D+02	3.3707D+02
6	3.2057D+02	3.1493D+02	3.1463D+02	3.1936D+02	3.3205D+02	3.5926D+02	3.7649D+02	3.8057D+02	3.5112D+02
7	3.2465D+02	3.1829D+02	3.1801D+02	3.2333D+02	3.3710D+02	3.6545D+02	3.8075D+02	3.7809D+02	3.4708D+02
8	3.2195D+02	3.2031D+02	3.2175D+02	3.2745D+02	3.3786D+02	3.5176D+02	3.5651D+02	3.5457D+02	3.0000D+02
9	3.1352D+02	3.1372D+02	3.1515D+02	3.1872D+02	3.2414D+02	3.2830D+02	3.2785D+02	3.0000D+02	3.0000D+02

EXHIBIT E-5

AVERAGE FUEL TEMPERATURE T = 2.0 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	4.9314D+02	4.4354D+02	4.4380D+02	4.8959D+02	6.0611D+02	8.4705D+02	9.6380D+02	8.9754D+02	6.7194D+02
2	4.4063D+02	4.3273D+02	4.4688D+C2	4.9107D+02	5.7227D+02	6.9334D+02	7.9391D+02	8.5924D+02	6.8340D+02
3	4.3424D+02	4.4032D+02	4.6145D+C2	5.0778D+02	5.8273D+02	6.7765D+02	7.8927D+02	9.1404D+02	7.3841D+02
4	4.6749D+02	4.7242D+02	4.9581D+02	5.5367D+02	6.5289D+02	7.8182D+02	9.3511D+02	1.1054D+03	8.7701D+02
5	5.5873D+02	5.3360D+02	5.5047D+02	6.2892D+02	7.8936D+02	1.0503D+03	1.2981D+03	1.4967D+03	1.1390D+03
6	7.5075D+02	6.2506D+02	6.1536D+02	7.1328D+02	9.8273D+02	1.5724D+03	1.9777D+03	2.1493D+03	1.5039D+03
7	8.3830D+02	6.9702D+02	6.8702D+02	7.9650D+02	1.0877D+03	1.7014D+03	2.0672D+03	2.0924D+03	1.4154D+03
8	7.7857D+02	7.4063D+02	7.6761D+02	8.8469D+02	1.1037D+03	1.4052D+03	1.5267D+03	1.5210D+03	3.0000D+02
9	5.9538D+02	5.9846D+02	6.2661D+02	7.0032D+02	8.1445D+02	9.0524D+02	9.0380D+02	3.0000D+02	3.0000D+02

EXHIBIT E-6

AVERAGE FUEL TEMPERATURE

T = 3.0 SEC

Y/X	1	2	3	4	5	6	7	8	9
1	5.5905D+02	4.9259D+02	4.9303D+02	5.5440D+02	7.1037D+02	1.0329D+03	1.1895D+03	1.1015D+03	7.9962D+02
2	4.8854D+02	4.7810D+02	4.9733D+02	5.5690D+02	6.6612D+02	8.2894D+02	9.6480D+02	1.0541D+03	8.1786D+02
3	4.7972D+02	4.8817D+02	5.1713D+02	5.8022D+02	6.8219D+02	8.1172D+02	9.6471D+02	1.1363D+03	8.9835D+02
4	5.2364D+02	5.3084D+02	5.6336D+02	6.4288D+02	7.7925D+02	9.5751D+02	1.1713D+03	1.4098D+03	1.0979D+03
5	6.4443D+02	6.1200D+02	6.3656D+02	7.4491D+02	9.6607D+02	1.3282D+03	1.6802D+03	1.9715D+03	1.4793D+03
6	8.9890D+02	7.3325D+02	7.2316D+02	8.5863D+02	1.2295D+03	2.0456D+03	2.6302D+03	2.9248D+03	2.0267D+03
7	1.0146D+03	8.2862D+02	8.1866D+02	9.7020D+02	1.3711D+03	2.2205D+03	2.7524D+03	2.8445D+03	1.9038D+03
8	9.3532D+02	8.8677D+02	9.2637D+02	1.0885D+03	1.3910D+03	1.8114D+03	1.9944D+03	2.0099D+03	3.0000D+02
9	6.9241D+02	6.9772D+02	7.3765D+02	8.3984D+02	9.9806D+02	1.1265D+03	1.1312D+03	3.0000D+02	3.0000D+02

BENCHMARK PROBLEM SOLUTION

Identification: 14-A1-2 Benchmark Problem ID.14-A1

Date Submitted: June 1976 By: H. Finnemann (KWU)

Date Accepted: June 1977 By: H.L. Dodds, Jr.(U. of Tenn.)
F.N. McDonnell (AECL-CRNL)

Descriptive Title: BWR Kinetics Benchmark Problem: 2-D Nodal Solution: Fifth Order Polynomial Expansion

Mathematical Model

The IQSBOX program solves the time-dependent two-group neutron diffusion equation in one, two or three dimensions by the nodal expansion method (NEM).

NEM is a consistent nodal technique that converges towards the exact solution of the diffusion equation for small mesh sizes. Subsidiary 1-D diffusion equations are solved in each box by polynomial expansion to obtain spatial coupling coefficients. Polynomials up to fifth order can be used. Time integration is performed by the backwards-difference algorithm combined with an exponential transformation technique.

Computer: CDC CYBER 175
Code: IQSBOX
Date Solved: January 1976 at Kraftwerk Union Erlangen

References

H. Finnemann
A Consistent Nodal Method for the Analysis of Space-Time Effects in Large LWR's.
Proc. of the Joint NEACRP/CSNI Specialists' Meeting on New Developments in Three-Dimensional Neutron Kinetics and Review of Kinetics Benchmark Calculations
Munich, January 22-24 (1975), MRR 145

F. Bennewitz, H. Finnemann, H. Moldaschl
Solution of the Multidimensional Neutron Diffusion Equation by Nodal Expansion
CONF-750413, Proc. Conf. on Comput.
Methods in Nucl. Eng., April 15 - 17, 1975
Charleston, South Carolina

F. Bennewitz, H. Finnemann, M.R. Wagner
Higher Order Corrections in Nodal Reactor Calculations.
Trans. Am. Nucl. Soc. 22, 250 (1975).

Results:

Uniform mesh with $\Delta x = \Delta y = 15$ cm (11 x 11 intervals)

1. Maximum eigenvalue for initial flux distribution:
$k_{eff} = .99631$

2. Exhibit A:
Normalized local power densities P_{ki} for initial flux distribution:
$$P_{ki} = \frac{\varepsilon}{V_k} \int_{V_k} (\Sigma f_1 \Phi_1 + \Sigma f_2 \Phi_2)\, dV / \bar{P}$$
Assembly k, V_k = Volume of Fuel
$k = 1,\ldots,78$

3. Maximum eigenvalue for configuration of withdrawn rod for cold reactor (feedback effects neglected):
$k_{eff} = 1.01531$

4. Exhibit B:
Average Power densitiy $\bar{P}$ versus time

5. Exhibit C:
Normalized Local Power Densities P_k and Average temperatures in volumes V_k, $k = 1,\ldots,78$, at $t = 0.4$ sec, 0.8 sec, 1.2 sec, 1.4 sec, 2.0 sec, 3.0 sec.

6. Maximum of $\bar{P} = 5.451 \cdot 10^3$; time of occurrence = 1.445 sec.

7. Exhibit D:
Average temperature $\bar{T} = \frac{1}{V_{core}} \int_{V_{core}} T(\vec{x},t)dV$ versus time

8. Number of unknowns in the problem: The unknowns are average node fluxes, average partial currents on the six surfaces of the node and average delayed precursor concentrations. The number of unknowns per mesh is given by $(2N+1)\cdot G + I$, where G is the number of energy groups, N the spatial deminsion, and I the number of delayed precursor groups. The number of unknowns is independent on the degree of the approximating polynomial.
The total number of unknowns for this problem is thus given by $(5\cdot 2 + 2) * 121 = 1452$
Number of time steps: 522 (automatic time-step selection such that local power change less than 12 % between time steps).

Computing time: 255 sec

EXHIBIT A

AVERAGE SUBASSEMBLY POWER (ABOVE) AND AVERAGE FUEL TEMPERATURE (BELOW)

9	.919 300.	.862 300.	.821 300.	.846 300.	.924 300.	.965 300.	.834 300.		
8	1.484 300.	1.285 300.	1.173 300.	1.221 300.	1.422 300.	1.675 300.	1.614 300.	1.311 300.	
7	1.672 300.	1.150 300.	.967 300.	1.022 300.	1.334 300.	2.054 300.	2.161 300.	1.614 300.	.834 300.
6	1.396 300.	.942 300.	.785 300.	.845 300.	1.150 300.	1.857 300.	2.054 300.	1.675 300.	.965 300.
5	.792 300.	.674 300.	.620 300.	.680 300.	.866 300.	1.150 300.	1.334 300.	1.422 300.	.924 300.
4	.515 300.	.493 300.	.495 300.	.555 300.	.680 300.	.845 300.	1.022 300.	1.221 300.	.846 300.
3	.416 300.	.409 300.	.427 300.	.495 300.	.620 300.	.785 300.	.967 300.	1.173 300.	.821 300.
2	.443 300.	.403 300.	.409 300.	.493 300.	.674 300.	.941 300.	1.150 300.	1.285 300.	.862 300.
1	.621 300.	.443 300.	.416 300.	.515 300.	.792 300.	1.396 300.	1.672 300.	1.484 300.	.919 300.
	1	2	3	4	5	6	7	8	9

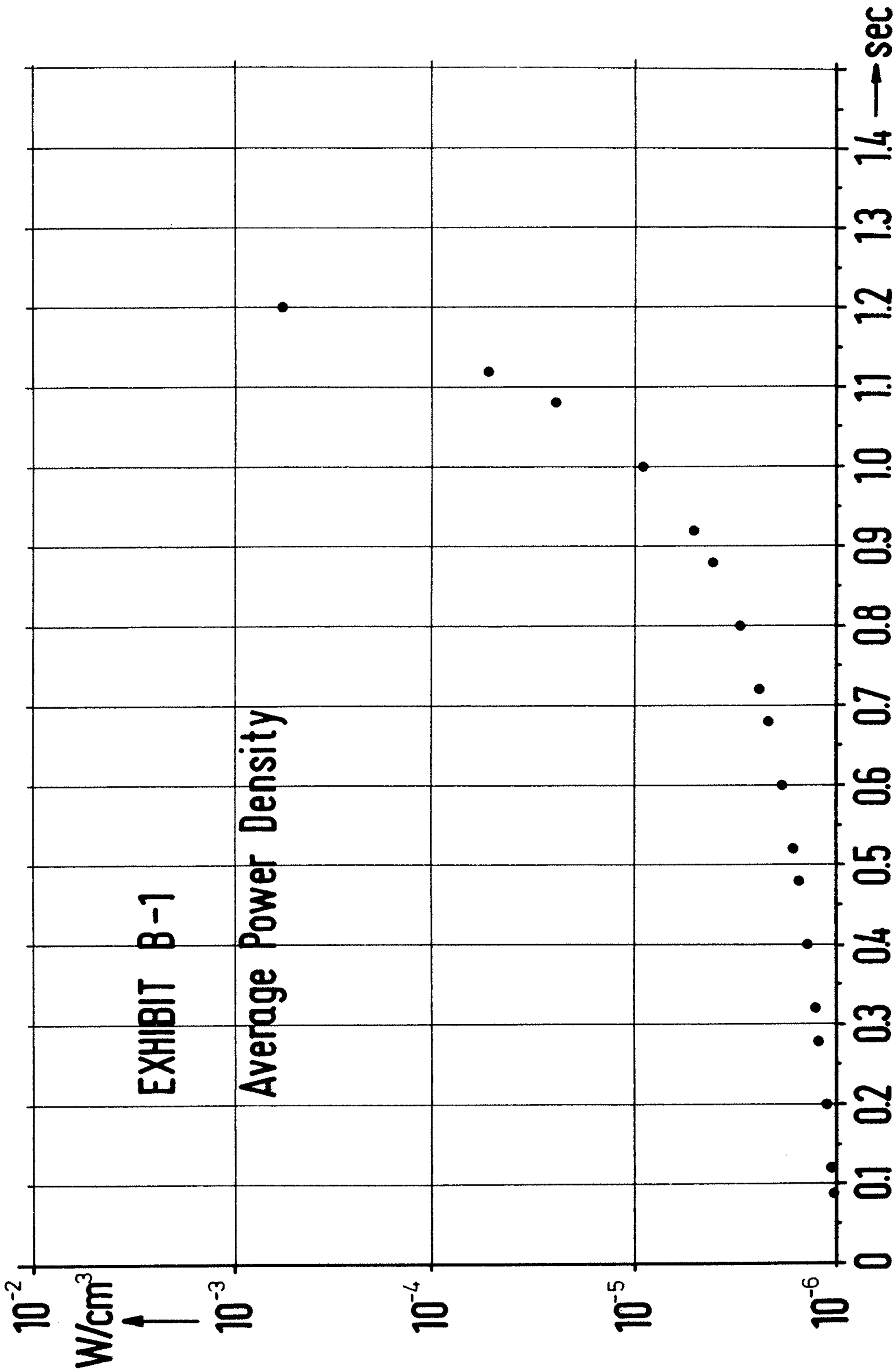
EXHIBIT B-1
Average Power Density
W/cm^3
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
sec

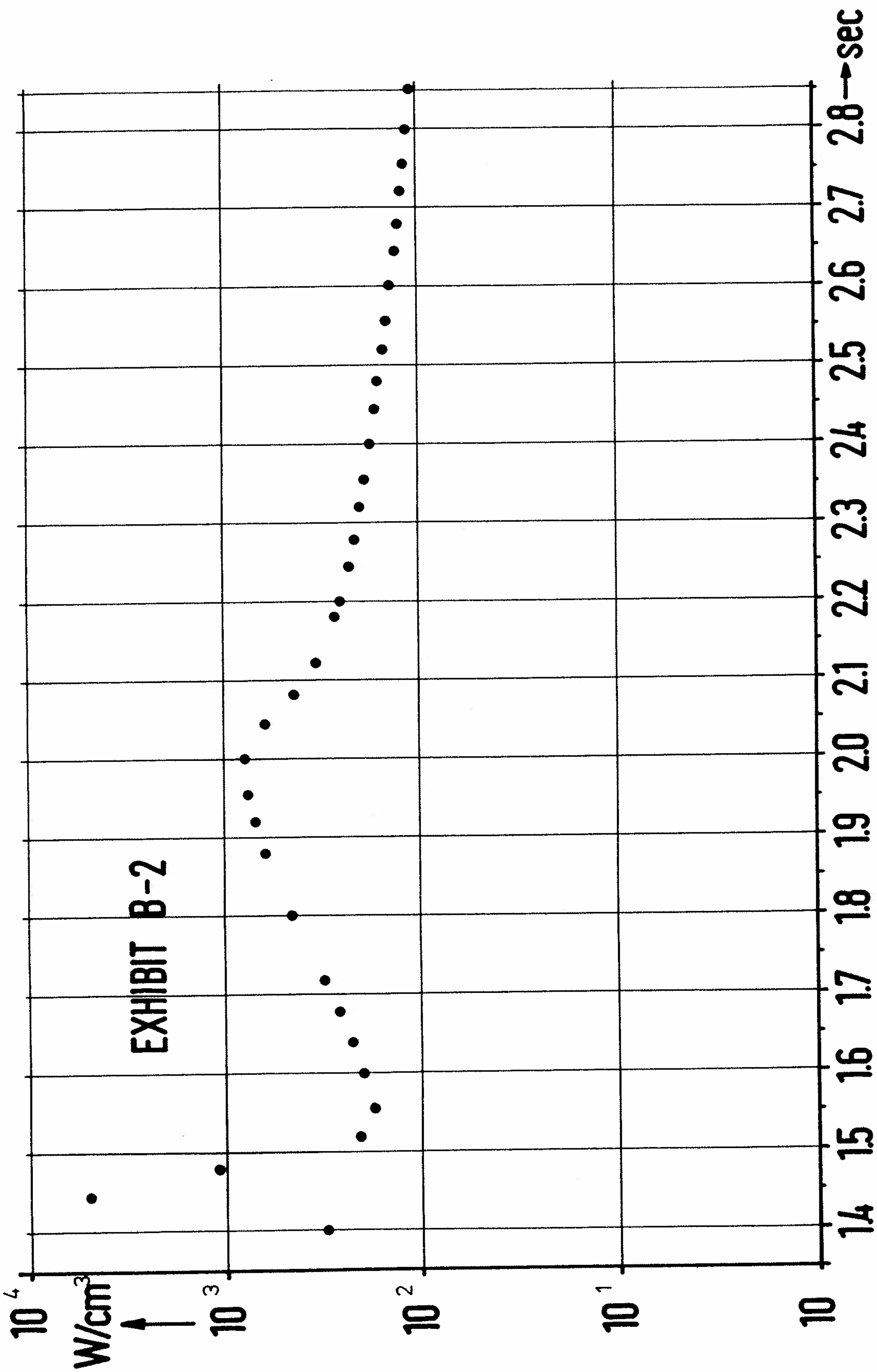
EXHIBIT B-2
10^4
W/cm^3
10^3
10^2
10^1
10
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
sec

EXHIBIT C-1 t = 0.4 sec

AVERAGE SUBASSEMBLY POWER (ABOVE) AND AVERAGE FUEL TEMPERATURE (BELOW)

9	.842 300.	.798 300.	.777 300.	.826 300.	.930 300.	.998 300.	.883 300.		
8	1.358 300.	1.188 300.	1.111 300.	1.194 300.	1.439 300.	1.744 300.	1.733 300.	1.461 300.	
7	1.529 300.	1.062 300.	.916 300.	1.003 300.	1.357 300.	2.154 300.	2.349 300.	1.881 300.	1.010 300.
6	1.276 300.	.869 300.	.744 300.	.830 300.	1.171 300.	1.948 300.	2.232 300.	1.945 300.	1.153 300.
5	.724 300.	.623 300.	.587 300.	.664 300.	.874 300.	1.192 300.	1.420 300.	1.558 300.	1.023 300.
4	.471 300.	.455 300.	.465 300.	.535 300.	.673 300.	.853 300.	1.048 300.	1.265 300.	.882 300.
3	.379 300.	.376 300.	.398 300.	.470 300.	.599 300.	.767 300.	.953 300.	1.163 300.	.816 300.
2	.401 300.	.368 300.	.378 300.	.462 300.	.638 300.	.896 300.	1.100 300.	1.235 300.	.831 300.
1	.560 300.	.402 300.	.382 300.	.480 300.	.743 300.	1.316 300.	1.581 300.	1.407 300.	.874 300.
	1	2	3	4	5	6	7	8	9

EXHIBIT C-2 t = 0.8 sec

AVERAGE SUBASSEMBLY POWER (ABOVE) AND AVERAGE FUEL TEMPERATURE (BELOW)

9	.734 300.	.708 300.	.715 300.	.796 300.	.938 300.	1.043 300.	.952 300.		
8	1.182 300.	1.053 300.	1.023 300.	1.157 300.	1.461 300.	1.841 300.	1.900 300.	1.673 300.	
7	1.328 300.	.939 300.	.844 300.	.976 300.	1.390 300.	2.294 300.	2.613 300.	2.260 300.	1.264 300.
6	1.107 300.	.768 300.	.686 300.	.809 300.	1.201 300.	2.075 300.	2.482 300.	2.326 300.	1.424 300.
5	.629 300.	.551 300.	.539 300.	.642 300.	.885 300.	1.251 300.	1.541 300.	1.736 300.	1.163 300.
4	.408 300.	.401 300.	.424 300.	.508 300.	.662 300.	.864 300.	1.084 300.	1.327 300.	.932 300.
3	.326 300.	.329 300.	.358 300.	.435 300.	.588 300.	.741 300.	.933 300.	1.148 300.	.810 300.
2	.342 300.	.318 300.	.335 300.	.419 300.	.588 300.	.888 300.	1.030 300.	1.164 300.	.787 300.
1	.475 300.	.345 300.	.386 300.	.430 300.	.675 300.	1.203 300.	1.452 300.	1.299 300.	.809 300.
	1	2	3	4	5	6	7	8	9

EXHIBIT C-3 t = 1.2 sec

AVERAGE SUBASSEMBLY POWER (ABOVE) AND AVERAGE FUEL TEMPERATURE (BELOW)

9	.596 300.	.593 300.	.634 300.	.755 300.	.941 300.	1.094 300.	1.035 300.		
8	.958 300.	.880 300.	.908 300.	1.102 300.	1.480 300.	1.954 300.	2.109 300.	1.946 300.	
7	1.073 300.	.782 300.	.751 300.	.938 300.	1.424 300.	2.461 300.	2.948 300.	2.771 300.	1.617 300.
6	.894 300.	.639 300.	.611 300.	.779 300.	1.233 300.	2.229 300.	2.801 300.	2.841 300.	1.798 300.
5	.509 300.	.459 300.	.479 300.	.612 300.	.895 300.	1.323 300.	1.696 300.	1.978 300.	1.348 300.
4	.330 300.	.334 300.	.372 300.	.473 300.	.647 300.	.875 300.	1.130 300.	1.406 300.	.996 300.
3	.261 300.	.270 300.	.307 300.	.391 300.	.523 300.	.705 300.	.904 300.	1.126 300.	.798 300.
2	.270 300.	.256 300.	.281 300.	.364 300.	.523 300.	.749 300.	.937 300.	1.070 300.	.727 300.
1	.370 300.	.275 300.	.277 300.	.367 300.	.587 300.	1.054 300.	1.281 300.	1.155 300.	.723 300.
	1	2	3	4	5	6	7	8	9

EXHIBIT C-4

t = 1.4 sec

AVERAGE SUBASSEMBLY POWER (ABOVE) AND AVERAGE FUEL TEMPERATURE (BELOW)

9	.539 302.	.543 302.	.596 302.	.731 303.	.934 304.	1.106 305.	1.064 304.		
8	.865 304.	.806 303.	.856 304.	1.071 305.	1.475 306.	1.988 308.	2.191 309.	2.069 309.	
7	.968 304.	.716 303.	.709 303.	.915 304.	1.428 306.	2.517 311.	3.091 313.	3.029 313.	1.803 308.
6	.807 303.	.586 302.	.578 302.	.762 303.	1.238 305.	2.283 310.	2.940 312.	3.103 313.	1.996 308.
5	.461 302.	.422 302.	.453 302.	.597 302.	.895 304.	1.349 306.	1.763 307.	2.091 309.	1.438 306.
4	.299 301.	.307 301.	.350 301.	.457 302.	.638 303.	.877 304.	1.147 305.	1.438 306.	1.023 304.
3	.236 301.	.247 301.	.286 301.	.371 302.	.509 302.	.687 303.	.889 304.	1.111 305.	.790 308.
2	.242 301.	.232 301.	.259 301.	.341 301.	.493 302.	.710 303.	.893 303.	1.023 304.	.697 303.
1	.329 301.	.247 301.	.254 301.	.341 301.	.548 302.	.985 304.	1.201 305.	1.086 305.	.682 303.
	1	2	3	4	5	6	7	8	9

EXHIBIT C-5 t = 2.0 sec

AVERAGE SUBASSEMBLY POWER (ABOVE) AND AVERAGE FUEL TEMPERATURE (BELOW)

	1	2	3	4	5	6	7	8	9
9	.461 595.	.471 596.	.529 622.	.667 692.	.877 800.	1.074 895.	1.080 880.		
8	.737 772.	.696 738.	.757 761.	.975 .872	1.386 1087.	1.942 1368.	2.266 1497.	2.309 1464.	
7	.825 829.	.618 689.	.628 682.	.836 788.	1.352 1062.	2.487 1655.	3.272 2001.	3.640 2048.	2.328 1373.
6	.692 742.	.508 619.	.514 611.	.701 708.	1.180 962.	2.269 1532.	3.125 1921.	3.722 2089.	2.537 1480.
5	.399 553.	.370 531.	.407 546.	.552 621.	.852 780.	1.331 1028.	1.830 1266.	2.296 1470.	1.641 1118
4	.264 466.	.273 469.	.318 492.	.422 548.	.602 645.	.849 775.	1.143 925.	1.475 1092.	1.073 869.
3	.212 433.	.223 438.	.261 458.	.341 503.	.472 577.	.645 673.	.846 783.	1.076 908.	.776 735.
2	.220 438.	.211 431.	.236 444.	.310 488.	.447 569.	.645 687.	.814 785.	.943 858.	.651 682.
1	.303 490.	.226 441.	.231 442.	.307 488.	.491 600.	.881 836.	1.074 954.	.977 892.	.619 673.

EXHIBIT C-6

t = 3.0 sec

AVERAGE SUBASSEMBLY POWER (ABOVE) AND AVERAGE FUEL TEMPERATURE (BELOW)

9	.502 714.	.503 717.	.553 756.	.677 858.	.871 1016.	1.053 1157.	1.050 1142.		
8	.802 963.	.746 916.	.789 952.	.987 1115.	1.371 1427.	1.894 1841.	2.192 2046.	2.225 2022.	
7	.900 1043.	.664 848.	.654 840.	.844 996.	1.332 1393.	2.419 2260.	3.157 2794.	3.497 2927.	2.236 1935.
6	.756 922.	.547 750.	.537 741.	.708 881.	1.163 1251.	2.208 2084.	3.018 2678.	3.579 2989.	2.439 2093.
5	.437 657.	.400 626.	.427 649.	.561 758.	.848 990.	1.303 1353.	1.776 1710.	2.222 2027.	1.589 1516.
4	.291 535.	.297 540.	.337 573.	.435 654.	.607 794.	.844 983.	1.127 1205.	1.450 1452.	1.055 1131.
3	.237 486.	.245 496.	.280 525.	.358 590.	.487 696.	.656 8 4.	.855 993.	1.083 1174.	.781 927.
2	.250 496.	.236 487.	.257 505.	.331 567.	.471 683.	.672 849.	.843 990.	.973 1095.	.670 845.
1	.346 570.	.255 501.	.254 502.	.331 567.	.521 725.	.929 1062.	1.128 1227.	1.021 1139.	.646 829.
	1	2	3	4	5	6	7	8	9

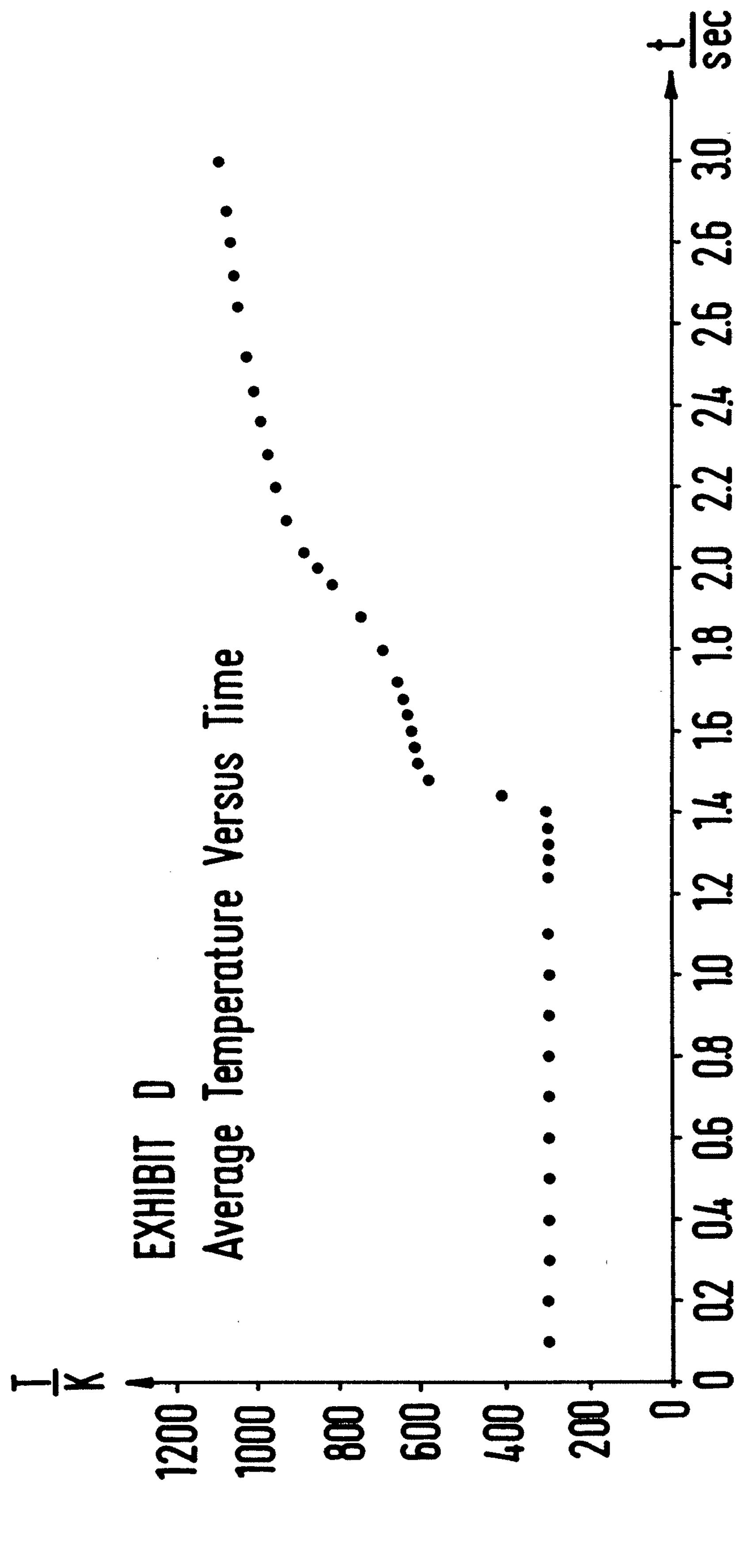
EXHIBIT D
Average Temperature Versus Time
T/K
1200
1000
800
600
400
200
0
t/sec
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.6
3.0