ANL-7416 Supplement 2
Mathematics and Computers
(UC-32)

ARGONNE CODE CENTER:
BENCHMARK PROBLEM BOOK

Prepared by the
Computational Benchmark Problems Committee of the
MATHEMATICS AND COMPUTATION DIVISION
OF THE AMERICAN NUCLEAR SOCIETY

Revised June 1977

Benchmark Problems Included

11. Multi-dimensional (x-y-z) LWR Model
13. Neutron Transport in a BWR Rod Bundle
14. Multi-dimensional (x-y-z) BWR Model
15. Neutronic Depletion Benchmark Problems

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ARGONNE NATIONAL LABORATORY
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Argonne, Illinois 60439

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BENCHMARK SOURCE SITUATION

Identification: 13

Date Submitted: November 1975 By: B. A. Zolotar (EPRI)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
W. A. Wittkopf (B&W)

Descriptive Title: Neutron Transport in a BWR Rod Bundle
7 x 7 BWR Fuel Assembly as Shown in Figure 1

Suggested Functions: Test Methods for Few-group, Two-dimensional Assembly Analysis

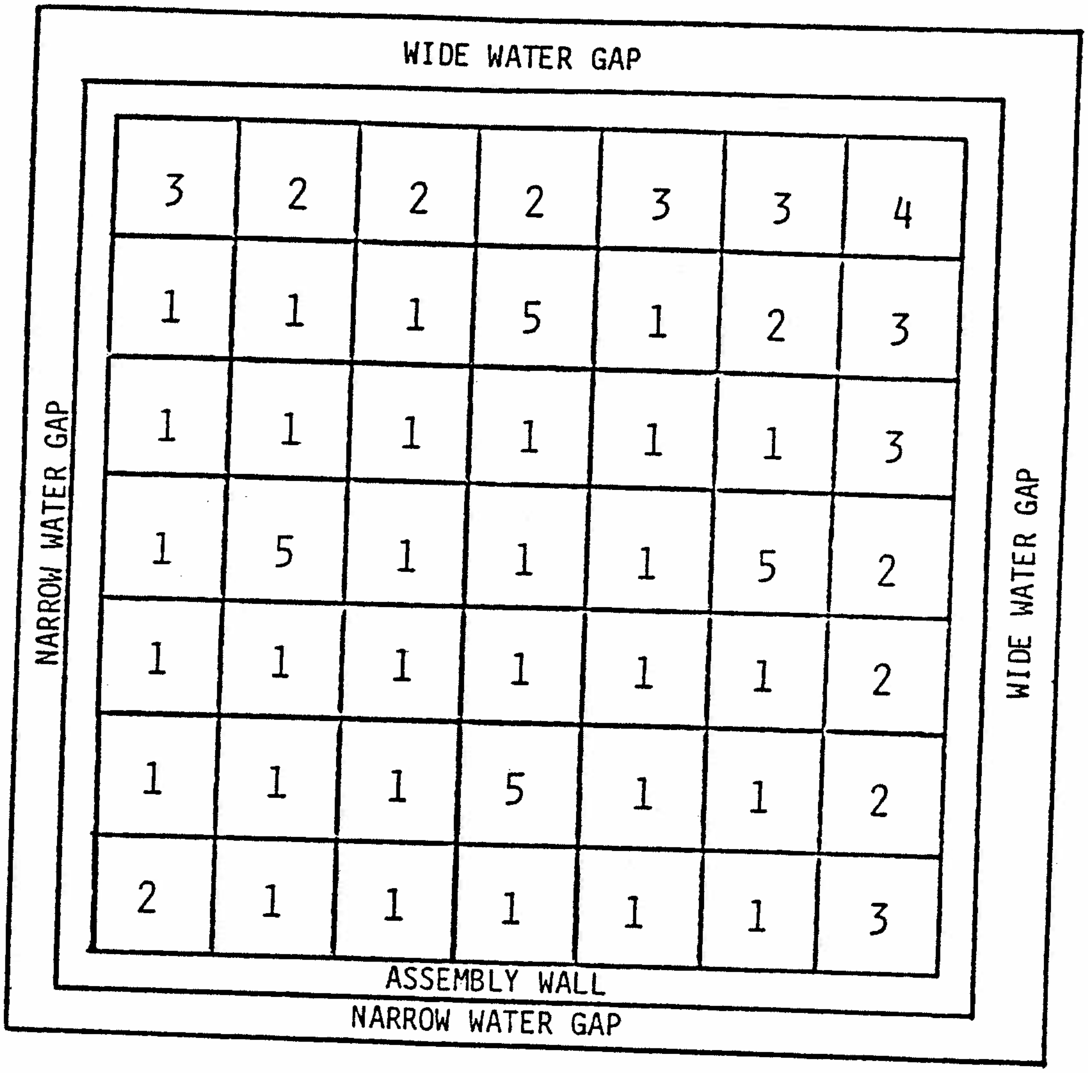

Figure 1. Bundle Diagram - Materials 1-4 represent fuel, material 5 contains fuel with poison, and the assembly wall is stainless steel.

BENCHMARK PROBLEM

Identification: 13-A1

Source Situation ID.13

Date Submitted: November 1975

By: B. A. Zolotar (EPRI)

Date Accepted: June 1977

By: H. L. Dodds, Jr. (U. of Tenn.)
W. A. Wittkopf (B&W)

Descriptive Title: Two-group, Two-dimensional (X-Y) Discrete Ordinates Model of a BWR Fuel Bundle as Shown in Figure 2

Reflective boundary conditions on external surfaces.

Two-Group Constants

Composition	Group i	$\Sigma_f(cm^{-1})$	$\Sigma_a(cm^{-1})$	$\nu\Sigma_f(cm^{-1})$	$\Sigma_T(cm^{-1})$	$\Sigma_{1\to2}(cm^{-1})$
1	1	2.281-3	8.983-3	5.925-3	2.531-1	1.069-2
	2	4.038-2	5.892-2	9.817-2	5.732-1	--
2	1	2.003-3	8.726-3	5.242-3	2.536-1	1.095-2
	2	3.385-2	5.174-2	8.228-2	5.767-1	--
3	1	1.830-3	8.587-3	4.820-3	2.535-1	1.112-2
	2	2.962-2	4.717-2	7.200-2	5.797-1	--
4	1	1.632-3	8.480-3	4.337-3	2.533-1	1.113-2
	2	2.428-2	4.140-2	5.900-2	5.837-1	--
5	1	2.155-3	9.593-3	5.605-3	2.506-1	1.016-2
	2	9.968-3	1.626-1	2.424-2	5.853-1	--
19	1	--	1.043-3	--	2.172-1	9.095-3
	2	--	4.394-3	--	4.748-1	--
20	1	--	1.983-4	--	2.476-1	3.682-2
	2	--	7.796-3	--	1.123+0	--

NOTE: Material 19 represents stainless steel; Material 20 represents water; Material 5 represents poison pins.

Expected Primary Results:
1. Group flux distribution
2. K_{eff}

Solutions: Solutions by DOT-III (13-A1-1) and TWØTRAN-II (13-A1-2)

20	20	20	20	20	20	20	20	20	20	20	20	0.47625
20	20	20	20	20	20	20	20	20	20	20	20	0.47625
20	19	19	19	19	19	19	19	19	19	20	20	0.34544
20	19	3	2	2	2	3	3	4	19	20	20	1.87452
20	19	1	1	1	5	1	2	3	19	20	20	1.87452
20	19	1	1	1	1	1	1	3	19	20	20	1.87452
20	19	1	5	1	1	1	5	2	19	20	20	1.87452
20	19	1	1	1	1	1	1	2	19	20	20	1.87452
20	19	1	1	1	5	1	1	2	19	20	20	1.87452
20	19	2	1	1	1	1	1	3	19	20	20	1.87452
20	19	19	19	19	19	19	19	19	19	20	20	0.34544
20	20	20	20	20	20	20	20	20	20	20	20	0.47498
0.47498	0.34544	1.87452	1.87452	1.87452	1.87452	1.87452	1.87452	1.87452	0.34544	0.47625	0.47625	

NOTE: All dimensions in centimeters.

Figure 2. Bundle Configuration and Material Assignments

BENCHMARK PROBLEM SOLUTION

Identification: 13-A1-1 Benchmark Problem ID.13-A1

Date Submitted: June 1976 By: B. A. Zolotar (EPRI)
F. J. Rahn (EPRI)

Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
W. A. Wittkopf (B&W)

Descriptive Title: Discrete Ordinate Solution

Computer: IBM-360, Model 195

Code: DOT III[1]

Date Solved: May 1976 at EPRI

References

1. W. A. Rhoades and F. R. Mynatt, The DOT III Two-Dimensional Discrete Ordinates Transport Code," ORNL-TM-4280, Oak Ridge National Lab. (1973).

Results

Discrete Ordinate Representation: S_8

Mesh Spacing: 4x4 = 16 meshes per region in configuration map

K_{eff} = 1.08714 Convergence criteria = 2×10^{-5}

Exhibit A: Benchmark Solution

Exhibit B: S_8 Angular Quadrature

Exhibit C: Mesh and Angular Quadrature Convergence of K_{eff}

Exhibit D: Mesh Convergence - Group 1 Fluxes

Exhibit E: Mesh Convergence - Group 2 Fluxes

Exhibit F: Angular Quadrature Convergence - Group 1 Fluxes

Exhibit G: Angular Quadrature Convergence - Group 2 Fluxes

.1767 .1115	.1757 .1108	.1764 .1065	.1793 .0998	.1792 .0957	.1781 .0953	.1755 .0991	.1721 .1069	.1660 .1183	.1629 .1260	.1629 .1283	.1625 .1303
.1779 .1081	.1774 .1066	.1788 .1015	.1819 .0944	.1820 .0903	.1808 .0900	.1783 .0939	.1747 .1018	.1680 .1137	.1642 .1222	.1638 .1256	
.1788 .1066	.1797 .1011	.1827 .0950	.1863 .0876	.1862 .0834	.1851 .0831	.1824 .0871	.1788 .0952	.1716 .1076	.1663 .1171		
.1840 .0945	.1864 .0902	.1924 .0827	.1970 .0748	.1964 .0703	.1944 .0690	.1922 .0741	.1891 .0828	.1805 .0960			
.1920 .0813	.1949 .0773	.2023 .0687	.2064 .0607	.2048 .0554	.1999 .0497	.2015 .0587	.1984 .0687				
.1954 .0733	.1980 .0693	.2048 .0606	.2080 .0520	.2082 .0490	.2071 .0483	.2055 .0518					
.1972 .0699	.1995 .0659	.2054 .0565	.2060 .0441	.2096 .0459	.2098 .0471						
.1976 .0706	.2000 .0667	.2066 .0580	.2100 .0495	.2103 .0466							
.1969 .0750	.1997 .0710	.2067 .0626	.2111 .0546								
.1928 .0834	.1951 .0790	.2014 .0708									
.1906 .0884	.1916 .0853										
.1910 .0903											

Group 1 Flux
Group 2 Flux

$K_{eff} = 1.08714$

BENCHMARK SOLUTION

S_8 - 4x4 Mesh Intervals

EXHIBIT A

ANGL	WEIGHT	ETA	MU
1	0.0	-0.2182178E+00	-0.9758998E+00
2	0.2665018E-01	-0.2182178E+00	-0.9511893E+00
3	0.2529317E-01	-0.2182178E+00	-0.7867955E+00
4	0.2529317E-01	-0.2182178E+00	-0.5773501E+00
5	0.2665018E-01	-0.2182178E+00	-0.2182178E+00
6	0.2665018E-01	-0.2182178E+00	0.2182178E+00
7	0.2529317E-01	-0.2182178E+00	0.5773501E+00
8	0.2529317E-01	-0.2182178E+00	0.7867955E+00
9	0.2665018E-01	-0.2182178E+00	0.9511893E+00
10	0.0	-0.5773501E+00	-0.8164963E+00
11	0.2529317E-01	-0.5773501E+00	-0.7867955E+00
12	0.1829034E-01	-0.5773501E+00	-0.5773501E+00
13	0.2529317E-01	-0.5773501E+00	-0.2182178E+00
14	0.2529317E-01	-0.5773501E+00	0.2182178E+00
15	0.1829034E-01	-0.5773501E+00	0.5773501E+00
16	0.2529317E-01	-0.5773501E+00	0.7867955E+00
17	0.0	-0.7867955E+00	-0.6172131E+00
18	0.2529317E-01	-0.7867955E+00	-0.5773501E+00
19	0.2529317E-01	-0.7867955E+00	-0.2182178E+00
20	0.2529317E-01	-0.7867955E+00	0.2182178E+00
21	0.2529317E-01	-0.7867955E+00	0.5773501E+00
22	0.0	-0.9511893E+00	-0.3086066E+00
23	0.2665018E-01	-0.9511893E+00	-0.2182178E+00
24	0.2665018E-01	-0.9511893E+00	0.2182178E+00
25	0.0	0.2182178E+00	-0.9758998E+00
26	0.2665018E-01	0.2182178E+00	-0.9511893E+00
27	0.2529317E-01	0.2182178E+00	-0.7867955E+00
28	0.2529317E-01	0.2182178E+00	-0.5773501E+00
29	0.2665018E-01	0.2182178E+00	-0.2182178E+00
30	0.2665018E-01	0.2182178E+00	0.2182178E+00
31	0.2529317E-01	0.2182178E+00	0.5773501E+00
32	0.2529317E-01	0.2182178E+00	0.7867955E+00
33	0.2665018E-01	0.2182178E+00	0.9511893E+00
34	0.0	0.5773501E+00	-0.8164963E+00
35	0.2529317E-01	0.5773501E+00	-0.7867955E+00
36	0.1829034E-01	0.5773501E+00	-0.5773501E+00
37	0.2529317E-01	0.5773501E+00	-0.2182178E+00
38	0.2529317E-01	0.5773501E+00	0.2182178E+00
39	0.1829034E-01	0.5773501E+00	0.5773501E+00
40	0.2529317E-01	0.5773501E+00	0.7867955E+00
41	0.0	0.7867955E+00	-0.6172131E+00
42	0.2529317E-01	0.7867955E+00	-0.5773501E+00
43	0.2529317E-01	0.7867955E+00	-0.2182178E+00
44	0.2529317E-01	0.7867955E+00	0.2182178E+00
45	0.2529317E-01	0.7867955E+00	0.5773501E+00
46	0.0	0.9511893E+00	-0.3086066E+00
47	0.2665018E-01	0.9511893E+00	-0.2182178E+00
48	0.2665018E-01	0.9511893E+00	0.2182178E+00

S_8 ANGULAR QUADRATURE

EXHIBIT B

Mesh convergence with S_8

1x1 K_{eff}=1.08441
2x2 K_{eff}=1.08709
4x4 K_{eff}=1.08714 (Benchmark Solution)

Angular Quadrature Convergence with 2x2 Mesh Interval

S_2 K_{eff}=1.09195
S_4 K_{eff}=1.08724
S_8 K_{eff}=1.08709

MESH AND ANGULAR QUADRATURE CONVERGENCE OF K_{eff}

EXHIBIT C

.1771 .1768 .1767	.1757 .1758 .1757	.1764 .1764 .1764	.1788 .1793 .1793	.1796 .1793 .1792	.1782 .1781 .1781	.1760 .1756 .1755	.1718 .1721 .1721	.1658 .1659 .1660	.1631 .1629 .1629	.1632 .1629 .1629	.1630 .1627 .1625
.1777 .1779 .1779	.1774 .1772 .1774	.1787 .1788 .1788	.1818 .1820 .1819	.1826 .1821 .1820	.1809 .1809 .1808	.1788 .1784 .1783	.1746 .1748 .1747	.1680 .1680 .1680	.1642 .1641 .1642	.1634 .1637 .1638	
.1785 .1785 .1788	.1792 .1796 .1797	.1824 .1835 .1827	.1858 .1863 .1863	.1867 .1863 .1862	.1852 .1850 .1851	.1827 .1825 .1824	.1785 .1789 .1788	.1716 .1715 .1716	.1649 .1664 .1663		
.1838 .1840 .1840	.1860 .1863 .1864	.1916 .1923 .1924	.1964 .1969 .1970	.1957 .1962 .1964	.1937 .1944 .1944	.1917 .1921 .1922	.1886 .1890 .1891	.1796 .1803 .1805			
.1919 .1921 .1920	.1946 .1950 .1949	.2018 .2023 .2023	.2075 .2065 .2064	.2052 .2050 .2048	.2014 .2001 .1999	.2018 .2016 .2015	.1996 .1984 .1984				
.1963 .1965 .1954	.1984 .1981 .1980	.2042 .2047 .2048	.2086 .2082 .2080	.2080 .2082 .2082	.2073 .2072 .2071	.2053 .2055 .2055					
.1975 .1973 .1972	.1999 .1996 .1995	.2046 .2054 .2054	.2074 .2062 .2060	.2095 .2096 .2096	.2114 .2100 .2098						
.1984 .1977 .1976	.2005 .2001 .2000	.2060 .2065 .2066	.2104 .2101 .2100	.2101 .2103 .2103							
.1967 .1970 .1969	.1992 .1997 .1997	.2061 .2066 .2067	.2120 .2112 .2111								
.1926 .1928 .1928	.1947 .1950 .1951	.2006 .2013 .2014									
.1907 .1904 .1906	.1907 .1916 .1916										
.1911 .1911 .1910											

1x1, 2x2, 4x4 } meshes per interval

MESH CONVERGENCE WITH S_8

GROUP 1 FLUXES

EXHIBIT D

1.1085 1.1136 1.1145	1.1040 1.1067 1.1075	1.0629 1.0651 1.0651	.9944 .9971 .9976	.9546 .9563 .9570	.9544 .9533 .9532	.9901 .9902 .9906	1.0686 1.0684 1.0685	1.1853 1.1836 1.1831	1.2602 1.2599 1.2602	1.2825 1.2828 1.2833	1.3006 1.3022 1.3027
1.0751 1.0802 1.0814	1.0618 1.0654 1.0661	1.0120 1.0153 1.0151	.9393 .9437 .9444	.9000 .9023 .9034	.9008 .8996 .8996	.9370 .9376 .9385	1.0161 1.0176 1.0179	1.1380 1.1374 1.1368	1.2212 1.2215 1.2221	1.2530 1.2548 1.2556	
1.0374 1.0416 1.0659	1.0078 1.0112 1.0113	.9531 .9507 .9503	.8741 .8760 .8757	.8358 .8345 .8343	.8418 .8305 .8306	.8750 .8710 .8713	.9532 .9528 .9522	1.0828 1.0770 1.0761	1.1678 1.1710 1.1708		
.9435 .9450 .9447	.9109 .9086 .9016	.8350 .8285 .8274	.7502 .7485 .7479	.7034 .7029 .7030	.6907 .6910 .6898	.7426 .7413 .7413	.8325 .8284 .8275	.9705 .9610 .9595			
.8072 .8119 .8128	.7717 .7734 .7731	.6893 .6877 .6871	.6080 .6076 .6073	.5480 .5543 .5544	.5068 .4973 .4965	.5822 .5871 .5871	.6898 .6874 .6867				
.7278 .7314 .7328	.6921 .6928 .6930	.6048 .6054 .6057	.5123 .5193 .5197	.4778 .4878 .4898	.4740 .4820 .4827	.5050 .5154 .5175					
.6975 .6986 .6991	.6663 .6587 .6592	.5632 .5657 .5649	.4479 .4413 .4409	.4503 .4578 .4587	.4802 .4704 .4705						
.7003 .7048 .7064	.6648 .6662 .6665	.5786 .5797 .5802	.4870 .4944 .4947	.4539 .4635 .4655							
.7424 .7489 .7501	.7068 .7102 .7102	.6260 .6260 .6258	.5448 .5462 .5463								
.8240 .8276 .8341	.7905 .7900 .7897	.7135 .7089 .7082									
.8793 .8832 .8844	.8484 .8521 .8529										
.8918 .8989 .9031											

1x1 }
2x2 } meshes per interval
4x4 }

MESH CONVERGENCE WITH S_8

GROUP 2 FLUXES x 10

EXHIBIT E

.1904 .1788 .1768	.1901 .1778 .1758	.1840 .1774 .1764	.1817 .1801 .1793	.1871 .1827 .1793	.1861 .1797 .1781	.1830 .1770 .1756	.1735 .1726 .1721	.1718 .1664 .1659	.1771 .1647 .1629	.1772 .1648 .1629	.1797 .1648 .1627
.1903 .1797 .1779	.1879 .1787 .1772	.1857 .1793 .1788	.1837 .1826 .1820	.1879 .1831 .1821	.1875 .1818 .1809	.1838 .1793 .1784	.1754 .1751 .1748	.1733 .1680 .1680	.1755 .1652 .1641	.1794 .1656 .1637	
.1884 .1797 .1785	.1897 .1808 .1796	.1876 .1825 .1835	.1867 .1861 .1863	.1889 .1867 .1863	.1897 .1852 .1850	.1846 .1827 .1825	.1786 .1785 .1789	.1757 .1710 .1715	.1771 .1670 .1664		
.1870 .1840 .1840	.1882 .1860 .1863	.1933 .1914 .1923	.1963 .1959 .1969	.1933 .1955 .1962	.1932 .1938 .1944	.1888 .1913 .1921	.1880 .1878 .1890	.1829 .1793 .1803			
.1908 .1926 .1921	.1925 .1949 .1950	.1990 .2015 .2023	.2052 .2055 .2065	.2023 .2047 .2050	.1937 .1987 .2001	.1986 .2013 .2016	.1979 .1973 .1984				
.1995 .1973 .1965	.1998 .1990 .1981	.2013 .2042 .2047	.2047 .2081 .2082	.2044 .2079 .2082	.2051 .2072 .2072	.2019 .2050 .2055					
.2022 .1990 .1973	.2033 .2006 .1996	.2042 .2051 .2054	.2003 .2064 .2062	.2069 .2097 .2096	.2103 .2110 .2100						
.2019 .1993 .1977	.2020 .2011 .2001	.2031 .2061 .2065	.2072 .2100 .2101	.2070 .2102 .2103		S_2 S_4 S_8					
.1949 .1978 .1970	.1972 .1999 .1997	.2042 .2061 .2066	.2098 .2103 .2112								
.1971 .1936 .1928	.1973 .1952 .1950	.2008 .2008 .2013									
.1980 .1919 .1904	.2025 .1934 .1916										
.2008 .1929 .1911											

ANGULAR QUADRATURE CONVERGENCE WITH 2x2 MESH INTERVAL

GROUP 1 FLUXES

EXHIBIT F

1.0650 1.1185 1.1136	1.0722 1.1161 1.1067	1.0488 1.0734 1.0651	.9679 1.0042 .9971	.9365 .9619 .9563	.9404 .9595 .9533	.9758 .9957 .9902	1.0501 1.0755 1.0684	1.1835 1.1923 1.1836	1.2547 1.2705 1.2599	1.2616 1.2937 1.2828	1.2615 1.3129 1.3022
1.0437 1.0825 1.0802	1.0397 1.0716 1.0654	1.0031 1.0213 1.0153	.9170 .9478 .9437	.8851 .9056 .9023	.8903 .9027 .8996	.9261 .9407 .9376	1.0007 1.0220 1.0176	1.1402 1.1437 1.1374	1.2162 1.2302 1.2215	1.2389 1.2628 1.2548	
1.0201 1.0458 1.0416	1.0004 1.0175 1.0112	.9472 .9557 .9507	.8626 .8789 .8760	.8247 .8378 .8345	.8327 .8337 .8305	.8673 .8748 .8710	.9477 .9560 .9528	1.0872 1.0825 1.0770	1.1726 1.1793 1.1710		
.9351 .9483 .9450	.9061 .9119 .9086	.8363 .8326 .8285	.7587 .7503 .7485	.7030 .7040 .7029	.7026 .6930 .6910	.7455 .7425 .7413	.8462 .8303 .8284	.9803 .9668 .9610			
.7793 .8145 .8119	.7551 .7754 .7734	.6463 .6890 .6877	.6199 .6065 .6076	.5652 .5531 .5543	.5001 .4987 .4973	.6003 .5859 .5871	.6927 .6866 .6874				
.7066 .7323 .7314	.6764 .6948 .6928	.6011 .6063 .6054	.5268 .5179 .5193	.4783 .4852 .4878	.4901 .4817 .4820	.5075 .5129 .5154					
.6751 .6996 .6986	.6481 .6609 .6587	.5690 .5676 .5657	.4382 .4418 .4413	.4632 .4577 .4578	.4803 .4716 .4704						
.6760 .7059 .7048	.6461 .6686 .6662	.5716 .5808 .5797	.4997 .4931 .4944	.4525 .4612 .4635		S_2 S_4 S_8					
.7194 .7516 .7489	.6833 .7127 .7102	.6278 .6277 .6260	.5519 .5451 .5462								
.8069 .8311 .8276	.7755 .7938 .7900	.7047 .7133 .7089									
.8407 .8860 .8832	.8246 .8587 .8521										
.8355 .8973 .8989											

ANGULAR QUADRATURE CONVERGENCE WITH 2x2 MESH INTERVAL

GROUP 2 FLUXES x 10

EXHIBIT G

BENCHMARK PROBLEM SOLUTION

Identification: 13-A1-2

Benchmark Problem ID.13-A1

Date Submitted: July 1976

By: A. N. Mallen (SRL)

Date Accepted: June 1977

By: H. L. Dodds, Jr. (U. of Tenn.)
W. A. Wittkopf (B&W)

Descriptive Title: Discrete Ordinate Solution

Computer: IBM-360, Model 195

Code: TWOTRAN-II[1]

Date Solved: June, 1976 at SRL

References

1. K. D. Lathrop, F. W. Brinkley, "TWOTRAN-II, An Interfaced, Exportable Version of the TWOTRAN Code for Two Dimensional Transport," LA-4848-MS, Los Alamos Scientific Laboratory (1973)

2. B. A. Zolotar, F. J. Rahn, A. Nero, "BWR Rod Bundle Benchmark Problem," Trans. of Am. Nucl. Soc. 23, 214 (1976)

Results:

Discrete Ordinate Representation: S_8

Mesh Spacing: 4x4 = 16 meshes per region in configuration map

Normalization = 1 source neutron; K_{eff} convergence criteria = 2×10^{-5}

$K_{eff} = 1.08727$

Exhibit A-1: Benchmark Solution

Exhibit A-2: Comparison with DOT-III Benchmark Solution

Exhibit B: S_8 Angular Quadrature

Exhibit C-1: Mesh and Angular Quadrature Convergence of K_{eff}

Exhibit C-2: Difference in Benchmark Solutions of K_{eff} {(DOT-III)-(TWOTRAN2)}

Exhibit D: Mesh Convergence - Group 1 Fluxes

Exhibit E: Mesh Convergence - Group 2 Fluxes

Exhibit F: Angular Quadrature Convergence - Group 1 Fluxes

Exhibit G: Angular Quadrature Convergence - Group 2 Fluxes

Exhibit H: TWOTRAN2 Execution Parameters

.1745 .1119	.1738 .1110	.1752 .1066	.1783 .0999	.1781 .0960	.1769 .0956	.1747 .0995	.1715 .1073	.1652 .1188	.1614 .1267	.1611 .1292	.1603 .1312
.1761 .1086	.1758 .1068	.1779 .1016	.1811 .0946	.1812 .0907	.1801 .0903	.1777 .0943	.1742 .1023	.1675 .1142	.1631 .1229	.1623 .1264	
.1775 .1045	.1787 .1012	.1822 .0950	.1860 .0876	.1860 .0836	.1848 .0832	.1823 .0875	.1789 .0955	.1715 .1079	.1655 .1175		
.1834 .0946	.1860 .0908	.1928 .0825	.1976 .0747	.1969 .0703	.1950 .0690	.1929 .0742	.1899 .0828	.1811 .0961			
.1914 .0815	.1948 .0774	.2029 .0686	.2071 .0606	.2055 .0554	.2004 .0496	.2023 .0588	.1994 .0687				
.1946 .0736	.1976 .0695	.2052 .0605	.2085 .0520	.2088 .0491	.2076 .0483	.2062 .0519					
.1961 .0701	.1989 .0660	.2058 .0564	.2062 .0440	.2099 .0459	.2099 .0469						
.1966 .0710	.1996 .0668	.2069 .0580	.2103 .0494	.2107 .0466							
.1961 .0752	.1992 .0711	.2070 .0624	.2115 .0545								
.1919 .0829	.1945 .0789	.2016 .0706									
.1890 .0887	.1904 .0854										
.1891 .0906											

S-8 (4X4) MESH GROUP 1
S-8 (4X4) MESH GROUP 2

BENCHMARK SOLUTION

S_8 - 4x4 Mesh Intervals

EXHIBIT A-1

.988 1.004	.989 1.002	.993 1.001	.994 1.001	.994 1.003	.993 1.003	.995 1.004	.997 1.004	.995 1.004	.991 1.006	.989 1.007	.986 1.007
.990 1.005	.991 1.002	.995 1.001	.996 1.003	.996 1.004	.996 1.003	.997 1.004	.997 1.005	.997 1.004	.993 1.006	.991 1.006	
.993 .980	.994 1.001	.998 1.000	.998 1.000	.999 1.002	.998 1.001	.999 1.005	1.001 1.004	.999 1.004	.995 1.003		
.997 1.001	.998 1.007	1.002 .998	1.003 .999	1.003 1.000	1.003 1.000	1.004 1.001	1.004 1.000	1.003 1.001			
.997 1.002	.999 1.001	1.003 .999	1.003 .998	1.003 1.000	1.003 .998	1.004 1.002	1.005 1.000				
.996 1.004	.998 1.003	1.002 .998	1.002 1.000	1.003 1.002	1.002 1.000	1.003 1.002					
.994 1.003	.990 1.002	1.002 .998	1.001 .998	1.001 1.000	1.000 .996						
.995 1.006	.998 1.001	1.001 1.000	1.001 .998	1.002 1.000			Group 1 Flux Group 2 Flux				
.996 1.003	.997 1.001	1.001 .997	1.002 .998								
.995 .994	.997 .999	1.001 .997									
.992 1.003	.994 1.001										
.990 1.003											

TWOTRAN2 K_{eff} = 1.08727

DOT-III K_{eff} = 1.08714

BENCHMARK FLUX RATIO

(TWOTRAN2/DOT-III)

S_8 - 4x4 Mesh Intervals

EXHIBIT A-2

ANGLE	MU	ETA	WEIGHT
1	0.192327	0.962299	0.291971
2	0.577350	0.793521	0.233138
3	0.192327	0.793521	0.233138
4	0.793521	0.577350	0.233138
5	0.577350	0.577350	0.225257
6	0.192327	0.577350	0.233138
7	0.962299	0.192327	0.291971
8	0.793521	0.192327	0.233138
9	0.577350	0.192327	0.233138
10	0.192327	0.192327	0.291971

S_8 ANGULAR QUADRATURE

EXHIBIT B

MESH CONVERGENCE WITH S_8

(1x1) K_{eff} = 1.08427

(2x2) K_{eff} = 1.08712

(4x4) K_{eff} = 1.08727

ANGULAR QUADRATURE CONVERGENCE WITH 2x2 MESH INTERVAL

S_2 K_{eff} = 1.09214

S_4 K_{eff} = 1.08708

S_8 K_{eff} = 1.08712

MESH AND ANGULAR QUADRATURE
CONVERGENCE OF K_{eff}

EXHIBIT C-1

MESH CONVERGENCE WITH S_8

(1x1) $\Delta K_{eff} = +0.00014$

(2x2) $\Delta K_{eff} = -0.00003$

(4x4) $\Delta K_{eff} = -0.00013$

ANGULAR QUADRATURE CONVERGENCE WITH 2x2 MESH INTERVAL

S_2 $\Delta K_{eff} = -0.00019$

S_4 $\Delta K_{eff} = +0.00016$

S_8 $\Delta K_{eff} = -0.00003$

$\Delta K_{eff} = K_{eff}$ (DOT-III) $- K_{eff}$ (TWOTRAN2)

DIFFERENCE IN K_{eff} SOLUTIONS

OF BENCHMARK PROBLEM

EXHIBIT C-2

.1751 .1747 .1745	.1736 .1738 .1738	.1751 .1751 .1752	.1778 .1783 .1783	.1785 .1781 .1781	.1768 .1770 .1769	.1750 .1746 .1747	.1711 .1715 .1715	.1648 .1651 .1652	.1613 .1613 .1614	.1617 .1610 .1611	.1607 .1605 .1603
.1762 .1762 .1761	.1760 .1757 .1758	.1775 .1779 .1779	.1811 .1811 .1811	.1816 .1812 .1812	.1801 .1801 .1801	.1781 .1777 .1777	.1742 .1743 .1742	.1672 .1674 .1675	.1630 .1629 .1631	.1616 .1623 .1623	
.1770 .1773 .1775	.1782 .1785 .1787	.1818 .1821 .1822	.1855 .1860 .1860	.1863 .1860 .1860	.1849 .1847 .1848	.1824 .1823 .1823	.1786 .1789 .1789	.1711 .1714 .1715	.1645 .1655 .1655		
.1831 .1834 .1834	.1855 .1859 .1860	.1918 .1926 .1928	.1971 .1975 .1976	.1962 .1968 .1969	.1940 .1950 .1950	.1922 .1928 .1929	.1893 .1899 .1899	.1801 .1809 .1811			
.1915 .1915 .1914	.1947 .1949 .1948	.2024 .2028 .2029	.2085 .2072 .2071	.2059 .2056 .2055	.2021 .2005 .2004	.2026 .2024 .2023	.2008 .1994 .1994				
.1955 .1946 .1946	.1980 .1976 .1976	.2045 .2051 .2052	.2092 .2086 .2085	.2084 .2088 .2088	.2077 .2076 .2076	.2058 .2062 .2062					
.1964 .1962 .1961	.1993 .1989 .1989	.2048 .2058 .2058	.2078 .2063 .2062	.2098 .2099 .2099	.2118 .2101 .2099						
.1974 .1966 .1966	.2001 .1996 .1996	.2062 .2068 .2069	.2109 .2104 .2103	.2103 .2107 .2107							
.1961 .1962 .1961	.1990 .1993 .1992	.2066 .2070 .2070	.2128 .2117 .2115								
.1916 .1919 .1919	.1941 .1944 .1945	.2006 .2014 .2016									
.1892 .1888 .1890	.1894 .1903 .1904										
.1895 .1893 .1891											

S-8 (1X1) MESH
S-8 (2X2) MESH
S-8 (4X4) MESH

MESH CONVERGENCE WITH S_8

GROUP 1 FLUXES

EXHIBIT D

.1113	.1106	.1064	.0996	.0957	.0956	.0993	.1072	.1188	.1264	.1289	.1308
.1118	.1108	.1066	.0999	.0959	.0956	.0994	.1073	.1188	.1266	.1291	.1312
.1119	.1110	.1066	.0999	.0960	.0956	.0995	.1073	.1188	.1267	.1292	.1312
.1079	.1064	.1013	.0941	.0903	.0903	.0940	.1019	.1140	.1226	.1258	
.1084	.1067	.1016	.0945	.0905	.0902	.0942	.1022	.1142	.1228	.1263	
.1086	.1068	.1016	.0946	.0907	.0903	.0943	.1023	.1142	.1229	.1264	
.1040	.1009	.0953	.0875	.0838	.0844	.0878	.0956	.1084	.1171		
.1044	.1012	.0950	.0877	.0836	.0832	.0874	.0956	.1080	.1176		
.1045	.1012	.0950	.0876	.0836	.0832	.0875	.0955	.1079	.1175		
.0945	.0910	.0833	.0749	.0703	.0690	.0743	.0832	.0970			
.0946	.0908	.0826	.0747	.0703	.0691	.0742	.0829	.0962			
.0946	.0908	.0825	.0747	.0703	.0690	.0742	.0828	.0961			
.0810	.0773	.0688	.0607	.0548	.0506	.0582	.0689				
.0814	.0774	.0686	.0606	.0554	.0496	.0588	.0688				
.0815	.0774	.0686	.0606	.0554	.0496	.0588	.0687				
.0731	.0695	.0605	.0512	.0479	.0474	.0506					
.0735	.0694	.0605	.0519	.0489	.0482	.0517					
.0736	.0695	.0605	.0520	.0491	.0483	.0519					
.0701	.0668	.0562	.0447	.0451	.0479						
.0701	.0660	.0565	.0440	.0458	.0469						
.0701	.0660	.0564	.0440	.0459	.0469						
.0704	.0667	.0578	.0487	.0455							
.0708	.0668	.0579	.0494	.0465							
.0710	.0668	.0580	.0494	.0466							
.0745	.0708	.0625	.0544								
.0751	.0711	.0625	.0545								
.0752	.0711	.0624	.0545								
.0826	.0791	.0712									
.0829	.0790	.0707									
.0829	.0789	.0706									
.0884	.0850										
.0886	.0853										
.0887	.0854										
.0899											
.0905											
.0906											

S-8 (1X1) MESH
S-8 (2X2) MESH
S-8 (4X4) MESH

MESH CONVERGENCE WITH S_8

GROUP 2 FLUXES

EXHIBIT E

.1904	.1901	.1840	.1817	.1872	.1862	.1831	.1737	.1719	.1773	.1774	.1798
.1764	.1756	.1756	.1788	.1792	.1780	.1757	.1717	.1649	.1624	.1623	.1621
.1747	.1738	.1751	.1783	.1781	.1770	.1746	.1715	.1651	.1613	.1610	.1605
.1903	.1879	.1857	.1837	.1879	.1876	.1839	.1755	.1735	.1756	.1796	
.1774	.1765	.1778	.1815	.1820	.1806	.1785	.1744	.1667	.1631	.1633	
.1762	.1757	.1779	.1811	.1812	.1801	.1777	.1743	.1674	.1629	.1623	
.1884	.1897	.1876	.1867	.1890	.1898	.1847	.1787	.1758	.1772		
.1778	.1790	.1813	.1855	.1860	.1846	.1824	.1782	.1700	.1652		
.1773	.1785	.1821	.1860	.1860	.1847	.1823	.1789	.1714	.1655		
.1869	.1882	.1933	.1963	.1934	.1933	.1889	.1881	.1831			
.1832	.1852	.1912	.1962	.1960	.1944	.1919	.1884	.1790			
.1834	.1859	.1926	.1975	.1968	.1950	.1928	.1899	.1809			
.1908	.1925	.1990	.2052	.2024	.1938	.1987	.1981				
.1923	.1949	.2021	.2065	.2056	.2008	.2022	.1983				
.1915	.1949	.2028	.2072	.2056	.2005	.2024	.1994				
.1995	.1998	.2012	.2047	.2044	.2052	.2020					
.1966	.1987	.2049	.2089	.2089	.2081	.2061					
.1946	.1976	.2051	.2086	.2088	.2076	.2062					
.2022	.2032	.2041	.2002	.2068	.2111						
.1980	.2000	.2055	.2070	.2105	.2115						
.1962	.1989	.2058	.2063	.2099	.2101						
.2018	.2019	.2030	.2071	.2070							
.1985	.2007	.2065	.2107	.2109							
.1966	.1996	.2068	.2104	.2107							
.1948	.1971	.2041	.2097								
.1973	.1997	.2065	.2112								
.1962	.1993	.2070	.2117								
.1970	.1972	.2007									
.1926	.1945	.2007									
.1919	.1944	.2014									
.1981	.2024										
.1904	.1921										
.1888	.1903										
.2007											
.1911											
.1893											

S-2 (2X2) MESH
S-4 (2X2) MESH
S-8 (2X2) MESH

ANGULAR QUADRATURE CONVERGENCE WITH 2x2 MESH INTERVAL

GROUP 1 FLUXES

EXHIBIT F

.1065 .1130 .1118	.1072 .1125 .1108	.1048 .1078 .1066	.0968 .1010 .0999	.0937 .0968 .0959	.0941 .0965 .0956	.0977 .1002 .0994	.1052 .1081 .1073	.1187 .1197 .1188	.1258 .1278 .1266	.1266 .1303 .1291	.1265 .1328 .1312
.1043 .1092 .1084	.1039 .1078 .1067	.1003 .1025 .1016	.0917 .0953 .0945	.0885 .0911 .0905	.0891 .0907 .0902	.0928 .0946 .0942	.1003 .1027 .1022	.1143 .1148 .1142	.1220 .1236 .1228	.1243 .1272 .1263	
.1020 .1052 .1044	.1000 .1021 .1012	.0947 .0957 .0950	.0863 .0881 .0877	.0825 .0841 .0836	.0833 .0835 .0832	.0869 .0878 .0874	.0950 .0959 .0956	.1090 .1084 .1080	.1176 .1183 .1176		
.0935 .0952 .0946	.0906 .0913 .0908	.0836 .0830 .0826	.0759 .0748 .0747	.0703 .0704 .0703	.0703 .0692 .0691	.0747 .0742 .0742	.0848 .0829 .0829	.0983 .0964 .0962			
.0779 .0820 .0814	.0755 .0779 .0774	.0696 .0687 .0686	.0620 .0604 .0606	.0565 .0552 .0554	.0500 .0496 .0496	.0601 .0585 .0588	.0707 .0684 .0688				
.0706 .0739 .0735	.0676 .0699 .0694	.0601 .0607 .0605	.0527 .0517 .0519	.0478 .0487 .0489	.0490 .0482 .0482	.0508 .0515 .0517					
.0675 .0706 .0701	.0648 .0664 .0660	.0569 .0568 .0565	.0438 .0441 .0440	.0463 .0458 .0458	.0480 .0471 .0469						
.0676 .0713 .0708	.0646 .0673 .0668	.0571 .0581 .0579	.0499 .0493 .0494	.0452 .0463 .0465							
.0705 .0759 .0751	.0683 .0717 .0711	.0627 .0627 .0625	.0551 .0544 .0545								
.0806 .0837 .0829	.0775 .0797 .0790	.0704 .0712 .0707									
.0840 .0896 .0886	.0824 .0865 .0853										
.0835 .0911 .0905											

S-2 (2X2) MESH
S-4 (2X2) MESH
S-8 (2X2) MESH

ANGULAR QUADRATURE CONVERGENCE WITH 2x2 MESH INTERVAL

GROUP 2 FLUXES

EXHIBIT G

S_n	Mesh	K_{eff} Convergence	CPU Time Min:Sec	Core Size Bytes
2	1x1	$2x10^{-5}$	:11	310K
4	1x1	$2x10^{-5}$	:13	310K
8	1x1	$2x10^{-5}$	:21	312K
2	2x2	$2x10^{-5}$	:21	319K
4	2x2	$2x10^{-5}$	:40	320K
8	2x2	$2x10^{-5}$	1:17	323K
2	4x4	$2x10^{-5}$	1:07	355K
4	4x4	$2x10^{-5}$	2:16	357K
8	4x4	$2x10^{-5}$	4:35	363K

TWOTRAN2 EXECUTION PARAMETERS

EXHIBIT H

BENCHMARK PROBLEM

Identification:	13-A2	Source Situation ID.13
Date Submitted:	January 1977	By: B. A. Zolotar (EPRI)
Date Accepted:	June 1977	By: H. L. Dodds, Jr. (U. of Tenn.) W. A. Wittkopf (B&W)

Descriptive Title: Diffusion Theory Problem for the LWR Bundle

This problem is defined precisely the same as Problem 13-A1 but with $D = \frac{1}{3\Sigma_T}$ (for each group) resulting in a two-group, two-dimensional diffusion theory problem.

Solutions:

1. Finite difference and linear finite-element: 13-A2-1

2. Finite difference: 13-A2-2

BENCHMARK PROBLEM SOLUTION

Identification: 13-A2-1

Benchmark Problem ID.13-A2

Date Submitted: January 1977

Date Accepted: June 1977

By: T. B. Fowler and D. R. Vondy (ORNL)

By: H. L. Dodds, Jr. (U. of Tenn.) W. A. Wittkopf (B&W)

Descriptive Title: Diffusion Theory Solution of LWR Bundle Problem

Computer: IBM-360/95

Code: VENTURE (Report ORNL-5062), mesh-centered finite difference

VANCER (Report to be published), mesh-edge finite difference

Results: The problem was solved with several uniform meshpoint arrangements assuming $D = \frac{1}{3\Sigma_T}$, and results normalized to one neutron source for the problem are displayed in Table 1. The VANCER code is not yet structured to solve small problems the most efficiently. For short execution times, the processor use time allocation is often more than reasonable under the multi-tasking system executing several jobs simultaneously. Fission source values by fueled assembly are shown in Table 2 for the mesh-centered formulation and in Table 3 for the mesh edge formulations; data is for horizontal rows of assemblies to the diagonal from the top down, normalized to one neutron source per unit volume in the fueled zone (unit average fission source). It should be noted that if the error from the difference formulation is assumed to be proportional to the square of the mesh spacing, there is little difference between these formulations for this problem (13^2 is somewhat $> 12^2$). The convergence level satisfied for these problems was 5×10^{-5} maximum relative flux change on outer iteration. Flux values for the 48 x 48 VENTURE mesh centered case are shown in Table 4 normalized to one source neutron (times 100).

Table 1. LWR Bundle Problem
Diffusion Theory Solutions

Meshpoints	Points Per Assembly	k_{eff}	Peak to Average Point Source	Peak Point Fast Flux	Processor Time* (min)
Mesh-centered finite-difference (VENTURE)					
12 x 12	1	1.09238	1.2543	0.1900	0.074
24 x 24	4	1.08759	1.3379	0.1907	0.102
48 x 48	16	1.08606	1.3851	0.1909	0.229
96 x 96	64	1.08565	1.4100	0.1910	0.921
Extrapolated	(∞)	1.0855	1.418	0.1910	
Mesh-edge finite-difference (VANCER) usual finite-difference (4 neighbors)					
13 x 13	(1)	1.08061	1.2607	0.1917	0.224
25 x 25	(4)	1.08389	1.3411	0.1912	0.477
49 x 49	(16)	1.08506	1.3861	0.1911	1.198
Linear finite-element (8 neighbors) (VANCER)					
13 x 13	(1)	1.08185	1.2659	0.1919	0.238
25 x 25	(4)	1.08454	1.3429	0.1913	0.500
49 x 49	(16)	1.08525	1.3869	0.1911	1.465

*Eigenvalue problem set up and solution; for the VENTURE solutions, the flux solution from the next smaller problem was used as a starting guess.

ID.13-A2-

Table 2. LWR Bundle Problem Assembly Fission Source
(Mesh-centered VENTURE code results)

Assembly (Horizontal row to diagonal, starts at top)	Mesh Points 12X12	24X24	48X48	96X96	Extrapolated
1	1.09074	1.09288	1.09363	1.09388	1.09396
2	1.14093	1.14285	1.14351	1.14372	1.14379
3	1.08737	1.08492	1.08435	1.08426	1.08423
4	1.06877	1.06881	1.06852	1.06841	1.06837
5	1.00338	1.00068	0.99999	0.99983	0.99978
6	1.09826	1.10063	1.10136	1.10157	1.10164
7	1.03213	1.03492	1.03581	1.03608	1.03617
8	1.25429	1.25888	1.26031	1.26072	1.26086
9	1.13426	1.13782	1.13862	1.13882	1.13889
10	1.05726	1.05700	1.05669	1.05655	1.05650
11	0.36380	0.37364	0.37668	0.37750	0.37777
12	1.10952	1.11015	1.11011	1.11004	1.11002
13	1.06466	1.06756	1.06812	1.06826	1.06830
14	1.13258	1.13063	1.13026	1.13022	1.13021
15	1.00320	1.00120	1.00048	1.00025	1.00017
16	0.96005	0.95104	0.94875	0.94820	0.94802
17	0.94115	0.93831	0.93728	0.93695	0.93684
18	1.00445	0.99613	0.99400	0.99350	0.99333
19	1.06608	1.06594	1.06565	1.06554	1.06550
20	0.34673	0.35498	0.35756	0.35825	0.35848
21	0.90326	0.89939	0.89814	0.89776	0.89763
22	0.91053	0.91070	0.91023	0.91006	0.91000
23	1.09183	1.08898	1.08838	1.08828	1.08825
24	0.96327	0.96071	0.95985	0.95958	0.95949
25	0.92118	0.91180	0.90943	0.90885	0.90866
26	1.15758	1.15953	1.16027	1.16051	1.16059
27	1.03817	1.03928	1.03946	1.03950	1.03951
28	1.08032	1.08162	1.08220	1.08241	1.08248

Table 3. LWR Bundle Problem Assembly Fission Source (Mesh-edge VANCER code results, 49X49 meshpoints)

Assembly	Usual Finite-Difference	Linear Finite-Element
1	1.09375	1.09424
2	1.14367	1.14399
3	1.08403	1.08412
4	1.06794	1.06824
5	0.99960	0.99954
6	1.10163	1.10157
7	1.03615	1.03609
8	1.26091	1.26123
9	1.13887	1.13908
10	1.05647	1.05633
11	0.37864	0.37822
12	1.11008	1.10975
13	1.06824	1.06821
14	1.13013	1.13024
15	1.00001	1.00001
16	0.94817	0.94750
17	0.93688	0.93650
18	0.99358	0.99272
19	1.06515	1.06558
20	0.35922	0.35893
21	0.89758	0.89734
22	0.90980	0.90992
23	1.08814	1.08834
24	0.95938	0.95939
25	0.90875	0.90824
26	1.16055	1.16100
27	1.03941	1.03975
28	1.08225	1.08291

Table 4. Average Flux Values x 100 for the 48 x 48 Mesh-Centered VENTURE Case by Zone (Fast and Thermal Values for the Map)

16.82 10.01	16.84 9.916	16.96 9.527	17.11 8.921	17.12 8.561	17.01 8.534	16.80 8.915	16.47 9.669	16.01 10.77	15.74 11.52	15.66 11.77	15.62 11.95
16.90 9.718	16.93 9.553	17.06 9.105	17.23 8.476	17.24 8.108	17.12 8.078	16.91 8.972	16.58 9.240	16.11 10.37	15.81 11.17	15.72 11.52	
17.01 9.335	17.06 9.090	17.22 8.612	17.41 7.956	17.41 7.580	17.29 7.546	17.08 7.955	16.75 8.739	16.25 9.901	15.92 10.72		
	→ Fuel ↓										
17.38 8.435	17.46 8.202	17.71 7.636	17.74 6.929	17.93 6.512	17.78 6.410	17.58 6.889	17.26 7.729	16.70 8.969			
17.94 7.216	18.03 6.966	18.33 6.350	18.56 5.616	18.52 5.134	18.30 4.793	18.21 5.469	17.89 6.400				
18.25 6.456	18.34 6.203	18.61 5.563	18.82 4.783	18.85 4.476	18.74 4.414	18.57 4.760					
18.40 6.119	18.48 5.862	18.72 5.175	18.86 4.205	18.99 4.167	18.95 4.241						
18.46 6.190	18.54 5.936	18.81 5.304	19.02 4.531	19.07 4.230							
18.39 6.589	18.48 6.338	18.78 5.731	19.03 5.001								
18.16 7.274	18.24 7.034	18.51 6.460									
17.98 7.765	18.04 7.534										
17.94 7.929											

BENCHMARK PROBLEM SOLUTION

Identification: 13-A2-2

Benchmark Problem ID.13-A2

Date Submitted: January 1977

By: B. A. Zolotar (EPRI)
F. J. Rahn (EPRI)

Date Accepted: June 1977

By: H. L. Dodds, Jr. (U. of Tenn.)
W. A. Wittkopf (B&W)

Descriptive Title: Diffusion Theory Solution with CITATION

Computer: IBM-360, Model 195

Code: CITATION

Date Solved: November 1975 at EPRI

Results:

K_{eff} = 1.087575

Mesh Spacing: 4 meshes per region in configuration map

Convergence criteria: eigenvalue 0.0001
flux 0.00004

Exhibit A: Group 1 Fluxes

Exhibit B: Group 2 Fluxes

.1679	.1681	.1692	.1707	.1709	.1697	.1676	.1642	.1597	.1571	.1563	.1558
.1687	.1689	.1703	.1719	.1720	.1708	.1687	.1654	.1607	.1579	.1569	
.1698	.1703	.1719	.1737	.1738	.1726	.1705	.1672	.1622	.1589		
.1735	.1743	.1769	.1793	.1792	.1777	.1757	.1725	.1668			
.1791	.1800	.1832	.1855	.1850	.1827	.1820	.1787				
.1822	.1831	.1861	.1880	.1883	.1872	.1856					
.1836	.1845	.1871	.1883	.1898	.1893						
.1843	.1851	.1880	.1901	.1906							
.1836	.1843	.1877	.1901								
.1813	.1820	.1850									
.1795	.1801										
.1791											

Exhibit A: Group 1 Fluxes

.1003	.0994	.0955	.0894	.0858	.0855	.0894	.0969	.1079	.1154	.1179	.1197
.0975	.0958	.0913	.0850	.0813	.0809	.0850	.0926	.1039	.1120	.1155	
.0936	.0909	.0863	.0796	.0759	.0755	.0797	.0892	.0992	.1072		
.0846	.0821	.0763	.0693	.0651	.0641	.0690	.0772	.0896			
.0724	.0698	.0635	.0562	.0514	.0474	.0548	.0639				
.0648	.0622	.0557	.0479	.0449	.0443	.0478					
.0615	.0588	.0518	.0415	.0418	.0425						
.0620	.0595	.0530	.0454	.0424							
.0661	.0636	.0573	.0500								
.0730	.0705	.0646									
.0778	.0754										
.0795											

Exhibit B: Group 2 Fluxes